PRIVATELY PROVIDED PUBLIC GOODS IN A DYNAMIC ECONOMY

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We show that when individuals can save (accumulate capital), they all eventually become public-good contributors. In steady state, larger economies have more contributors. If the public good is normal, then its quantity increases in population size in the open-loop equilibrium, but not necessarily in the feedback equilibrium. If both private and public goods are normal, then the open-loop equilibrium exhibits greater steady-state public provision than the feedback equilibrium. If private consumption is inferior the opposite is true. Interpreting individuals as countries, our findings suggest that all countries over time will become contributors toward a global public good.

Keywords: private provision, public goods, dynamic, intertemporal, differential game.

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1. INTRODUCTION

Voluntary public goods provision has been analysed mostly in static economies (see Cornes and Sandler (1983, 1996), and Bergstrom, Blume, and Varian (1986)). In the benchmark framework, individuals differ in their endowments and choose to spend on private consumption and on a contribution to the public good. The amount of the public good provided depends on the sum of the individuals' contributions. Consequently, an individual's incentive to provide depends on the decisions of other individuals. Common results, see Andreoni (1988) and Bergstrom, Blume, and Varian (1986), are that in the Nash equilibrium of the contribution game:

1. Wealthier individuals contribute and the less wealthy typically not.
2. As economies grow large (in terms of population) the number of contributors shrinks.
3. In the limit, as population tends to infinity, only the richest individual contributes.
4. If the public good is a normal good, then it's size increases with population size (converging to a finite limit), and if private consumption is normal, then each individual's contribution decreases with population size.

Result 1 is the classical free-rider problem. Results 2-3 imply that the free-rider problem gets worse for larger populations. Result 4 implies that a larger economy cannot end up with a smaller amount of the public good if the public good is normal.

Sofar, dynamic analysis of voluntary public-goods provision has been confined to models where the public good can be accumulated but where (identical) individuals receive the same period-by-period income and are assumed not to save. Voluntary contribution is modelled as a differential game, under different commitment assumptions. The equilibrium where each individual at date zero chooses a time path of present and future contributions to the public good, taking all other individuals' contribution paths as given, is referred to as the open loop Nash equilibrium (see Basar and Olsder (1982)). If individuals could revise their contribution plans in the future, they would (generally) no longer find their plans optimal.[1] If the assumption of commitment is relaxed, individuals will at each date play Nash in contribution levels. Individuals' contributions will be functions of the relevant state. Most of the literature focuses on Markov-perfect strategies (i.e. strategies being a function of only the

[1] This is due to the time-inconsistency problem, discovered by Kydland and Prescott (1977).

physical state variables).[2] The equilibrium confining to Markov-perfect strategies is usually referred to as the feedback Nash equilibrium (Basar and Olsder (1982)).

Within this framework Fershtman and Nitzan (1991) show (for linear-quadratic objective) that the open-loop Nash equilibrium (i.e. under commitment) yields a steady-state contribution level smaller than the Pareto-efficient one. They also show that the feedback Nash equilibrium (i.e. Markov perfect) yields a steady-state contribution level smaller than the one in the open-loop Nash equilibrium.

Itaya and Shimomura (2001) generalise the analysis of Fershtman and Nitzan (1991) to more general objectives. They focus on the relation between static public good games and dynamic. They show that the steady state open-loop equilibrium condition almost coincide with the first-order conditions of a static game under Nash conjectures. They also show that the steady state feedback equilibrium condition almost coincide with the conditions of a static game under general conjectural variations. They argue that general conjectural variations in static games can be viewed as steady-state feedback equilibria of dynamic games.

In our paper we will extend the voluntary public goods provision model to a dynamic (Ramsey) economy, where heterogeneous individuals can accumulate wealth (capital) over time. Individuals differ in their initial capital endowments and decide how much to consume privately and how much to contribute to the public good, at each point in time. In this way, the income of an individual becomes endogenous, and a strategic decision variable itself. The aim of the paper is to solve for the steady states of both the open-loop Nash and feedback equilibria to the dynamic contribution game, and contrast the equilibria to that of the static model.

It is shown that some of the results from the static model do not carry over to the dynamic one. In particular, in the steady state, the number of contributors increases proportionately with population size. However, each individual’s contribution declines if private consumption is normal. In the open-loop Nash equilibrium, total public goods provision is increasing in population size, but tends to a limit (as in the static model). In the feedback Nash equilibrium, however, the total provision may decline in population size.

Our results have implications for provision of global public goods. Individual contributors may be interpreted as individual countries contributing to a global public good

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[2] With strategies being functions of the history of the game, one can in most cases find trigger strategies to sustain the open-loop Nash equilibrium.

(such as the environment). If countries act strategically and there is free mobility of at least one of the production factors, then the countries will converge in capital ownership and all countries contribute equally toward the global public good.

2. THE ECONOMY

There are L individuals. At time t, individual i supplies her capital, $k^i(t)$, and one unit of labour (exogenously) on the market, and chooses her own private consumption level, $c^i(t)$, and how much to provide towards the public good, $g^i(t)\geq 0$. The rental rates of capital and labour are their marginal products, $f_K(K(t),L)$ and $f_L(K(t),L)$, respectively, where subscripts denote partial derivatives. The production technology exhibits constant returns-to-scale. The change in her capital stock is therefore

$$\dot{k}^i(t) = f_K(K(t),L)k^i(t) + f_L(K(t),L) - c^i(t) - g^i(t) \tag{1}$$

Individual i chooses time paths of her private and public consumption so as to maximise

$$\int_0^\infty e^{-\theta t} u\left(c^i(t), G(t)\right) dt \tag{2}$$

where $G(t)$ is the aggregate amount of the public good at time t, and J is the value function.

3. OPEN-LOOP NASH EQUILIBRIUM

At date 0, each individual chooses a time path of private consumption and own public provision (and thereby the time path of her capital stock) taking as given the decisions of all other individuals. Thus, in the open-loop Nash equilibrium, it is assumed that individuals at date 0 can commit to their future consumption and provision levels. If the individuals could re-optimise in the future they would generally not find their initial plan optimal (the reason is time-inconsistency in the Kydland and Prescott sense). We will solve for the time-consistent equilibrium in Section 4 (the closed-loop Nash equilibrium).

Denote with superscript $-i$ the variables without individual i's quantities, that is[3]

[3] Equivalent to choosing $c^i(t)$, $g^i(t)$ taking $\{c^j(t)\}^{j\neq i}$, $G^{-i}(t)$ as given (and taking $k^i(t)$, $\{k^j(t)\}^{j\neq i}$ as residually determined) is to choose $k^i(t)$, $g^i(t)$ taking $\{k^j(t)\}^{j\neq i}$, $G^{-i}(t)$ (and taking $\{c^j(t)\}^{j\neq i}$, $K^{j\neq i}(t)$ as residually determined). Under the two equivalent formulations the initial capital stocks $k^i(0)=k^i_0$ and $\{k^j(0)\}^{j\neq i}=\{k^j_0\}^{j\neq i}$ are taken as given. In the open-loop Nash equilibrium only the aggregate capital stock matters $K^{-i}(t)$, while in the feedback equilibrium one has to keep track of the distribution $\{k^j(t)\}^{j\neq i}$.

$$K^{-i}(t) = K(t) - k^i(t)$$
$$G^{-i}(t) = G(t) - g^i(t) \tag{3}$$

This is an ordinary optimal control problem where the functions $K^{-i}(t)$, $G^{-i}(t)$ and are taken as given for $t\varepsilon[0,\infty)$. The (current-value) Hamiltonian is

$$\begin{aligned} H^i(t) &= u\left(c^i(t), G^{-i}(t) + g^i(t)\right) \\ &+ q^i(t)\left\{f_K\left(K^{-i}(t) + k^i(t), L\right)k^i(t) + f_L\left(K^{-i}(t) + k^i(t), L\right) - c^i(t) - g^i(t)\right\} \\ &+ \nu(t) g^i(t) \end{aligned} \tag{4}$$

where $\nu^i(t)$ is the multiplier associated with the constraint $g^i(t)\geq 0$.[4]
The first-order conditions are

$$\frac{\partial H^i(t)}{\partial c^i(t)} = u_c\left(c^i(t), G(t)\right) - q^i(t) = 0 \tag{5}$$

$$\frac{\partial H^i(t)}{\partial g^i(t)} = u_G\left(c^i(t), G(t)\right) - q^i(t) + \nu^i(t) = 0 \tag{6}$$

$$\frac{\partial H^i(t)}{\partial k^i(t)} = q^i(t)\left\{f_K(K(t),L) + f_{KK}(K(t),L)k^i(t) + f_{LK}(K(t),L)\right\} = \theta q^i(t) - \dot{q}^i(t) \tag{7}$$

The initial value of the co-state, i.e. $q^i(0)$, is the minimum value that satisfies the individual's intertemporal budget constraint [equation (1) integrated].

Since, by constant returns-to-scale, $f_{LK} = -f_{KK}K/L$ we may write (7) as

$$\dot{q}^i(t) = \left[\theta - f_K(K(t),L) - f_{KK}(K(t),L)\left(k^i(t) - \bar{k}(t)\right)\right]q^i(t) \tag{8}$$

For an individual that finds contribution toward the public good optimal, the inequality constraint on $g^i(t)$ does not bind, and consequently $\nu^i(t)=0$. Combining (5) and (6) implies that the private consumption quantities of contributors are equalised. Thereby, by (5) [or (6)], the co-states are equal among contributors, i.e.

$$u_c\left(c^i(t), G(t)\right) = u_G\left(c^i(t), G(t)\right) = q^i(t) \tag{9}$$

For individuals that do not contribute, the following holds

[4] An individual with a "small" k^i_0 may find it optimal to take out from the public pool and use the part of the public good as private consumption (i.e. $g^i < 0$). The non-negativity constraint will bind for such an individual and forces $g^i=0$.

$$u_c\left(c^i(t), G(t)\right) \;=\; q^i(t) \;>\; u_G\left(c^i(t), G(t)\right) \tag{10}$$

Thus, non-contributors have higher co-states, i.e. higher marginal utility of assets, i.e. lower initial capital stocks.

Over time, individuals with lower capital (i.e. higher co-states) would have a lower growth in her co-state [follows from equation (8)]. The opposite is true for individuals with higher capital endowments. Consequently the co-states (and capital) converge over time, until all individuals own the average capital stock and all individuals are contributors.

The reason is that given the savings choice of the other individuals, an individual can marginally affect the future returns to savings and labour by strategically choosing the savings level. An individual with a small capital stock will earn relatively less from capital income than wage income, and has an incentive of increasing the return to labour and lowering the return to capital. This is done by marginally increasing her capital stock, which will increase the wage rate and lower the interest rate. This incentive is always there for an individual with a smaller than average capital stock. The opposite holds for an individual with a larger than average capital stock. Consequently, individuals will converge in their capital stocks.

The steady-state equilibrium is found as follows. Aggregate the individuals' capital-accumulation equations, and set the time derivatives of capital to zero, to obtain

$$f(K^*, L) \;=\; L\bar{c}^* \;+\; G^* \tag{11}$$

where $\bar{c}^*$ is the per-capita consumption level at steady state (equal for all individuals). The co-states being constant over time (and equalised steady state distribution of capital) gives [equation (8)]

$$\theta \;=\; f_K\left(K^*, L\right) \tag{12}$$

Consequently (by constant returns-to-scale) the aggregate capital-labour ratio is a function of the rate of time preference, say $K^*/L^* = \kappa(\theta)$. Dividing (11) by L and inserting the steady-state capital-labour ratio into (9) gives

$$u_c\left(f(\kappa(\theta), 1) - G^*/L,\, G(t)\right) \;=\; u_G\left(f(\kappa(\theta), 1) - G^*/L,\, G(t)\right) \tag{13}$$

We then have the following:

Proposition 1. *In the open-loop Nash equilibrium the following holds*:

1. *In the long run (steady state) all individuals will individually contribute the same amount*

towards the public good, regardless the level of capital they started with.

2. *The steady state of a larger economy (larger L) will have greater aggregate public goods provision if (and only if) the public good is a normal good, and a smaller per capita provision if (and only if) private consumption is normal.*

3. *The steady state number of contributors increase one-for-one with population size.*

The proof of 2 follows by differentiating (13).

Thus, as the economy gets large, more and more individuals contribute individually less and less. We will now relax the assumption about commitment.

4. FEEDBACK NASH EQUILIBRIUM

When individuals can re-optimise, they are not bound by their initially optimal plans. We therefore have to solve for the individuals' decisions recursively. As is common in general dynamic games, there are many equilibria, depending on what is assumed about the history of the game. We will rule out history-dependent strategies, and focus on Markov-perfect equilibria, i.e. allowing strategies as functions of the vector of individual capital stocks, denoted $\vec{k}(t)$. In the differential-game literature, this is referred to as the feedback Nash equilibrium (see Basar and Olsder (1982)). The individuals' strategies take the form :

$$c^i = c^i\left(\vec{k}(t)\right), \quad \boldsymbol{g}^i = \boldsymbol{g}^i\left(\vec{k}(t)\right) \tag{14}$$

At each date *t*, each individual chooses private consumption and own public provision (and thereby the change in her capital stock) taking as given the date-*t* decisions of all other individuals. In contrast to the open-loop equilibrium (commitment), an individual can affect the other individuals' future decisions by changing the future state (by equation (14)). This adds a strategic incentive to the savings decision of each individual (that is not present in the open-loop formulation).

Denote as $\vec{k}^{-i}(t)$ the vector of all individuals' capital stocks, excluding the one of individual *i*. The value function of individual *i* is a function of the state variables, as well as of time:

$$J\left(t, k^i(t), \vec{k}^{-i}(t)\right) = \max \int_t^\infty e^{-\theta\tau}\, u\left(c^i(\tau), G(\tau)\right) d\tau \tag{15}$$

At each date t, individual i solves the Hamilton-Jacobi-Bellman equation

$$-J_t\left(t, k^i(t), \vec{k}^{-i}(t)\right) = \max_{c^i(t), g^i(t)} \left\{ e^{-\theta t} u\left(c^i(t), G^{-i}(t) + g^i(t)\right) + J_{k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) \dot{k}^i(t) + \sum_{j \neq i} J_{k^j(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) \dot{k}^j(t) \right\} \tag{16}$$

where

$$\dot{k}^i(t) = f_K\left(K^{-i}(t) + k^i(t), L\right) k^i(t) + f_L\left(K^{-i}(t) + k^i(t), L\right) - c^i(t) - g^i(t) \tag{17}$$

$$\dot{k}^j(t) = f_K\left(K^{-i}(t) + k^i(t), L\right) k^j(t) + f_L\left(K^{-i}(t) + k^i(t), L\right) - x^j\left(k^i(t), \vec{k}^{-i}(t)\right) \tag{18}$$

$$x^j\left(k^i(t), \vec{k}^{-i}(t)\right) \equiv c^j\left(k^i(t), \vec{k}^{-i}(t)\right) + g^j\left(k^i(t), \vec{k}^{-i}(t)\right) \tag{19}$$

The first-order conditions are

$$e^{-\theta t} u_c\left(c^i(t), G(t)\right) - J_{k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) = 0 \tag{20}$$

$$e^{-\theta t} u_G\left(c^i(t), G(t)\right) - J_{k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) + \chi^i(t) = 0 \tag{21}$$

and the envelope conditions are

$$\begin{aligned} -J_{tk^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) &= J_{k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right)\left[f_K(K(t), L) + f_{KK}(K(t), L) k^i(t) + f_{LK}(K(t), L)\right] \\ &+ \sum_{j \neq i} J_{k^j(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right)\left[f_{KK}(K(t), L) k^j(t) + f_{LK}(K(t), L) - x^j_{k^i(t)}(\vec{k}(t))\right] \\ &+ J_{k^i(t) k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) \dot{k}^i(t) + \sum_{j \neq i} J_{k^j(t) k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) \dot{k}^j(t) \\ &+ e^{-\theta t} u_G\left(c^i(t), G(t)\right) \sum_{j \neq i} g^j_{k^i(t)}(\vec{k}(t)) \end{aligned} \tag{22}$$

$$\begin{aligned} -J_{tk^s(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) &= J_{k^i(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right)\left[f_{KK}(K(t), L) k^i(t) + f_{LK}(K(t), L)\right] \\ &+ \sum_{j \neq i} J_{k^j(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right)\left[f_{KK}(K(t), L) k^j(t) + f_{LK}(K(t), L) - x^j_{k^s(t)}(\vec{k}(t))\right] \\ &+ J_{k^s(t)} f_K(K(t), L) \\ &+ J_{k^i(t) k^s(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) \dot{k}^i(t) + \sum_{j \neq i} J_{k^j(t) k^s(t)}\left(t, k^i(t), \vec{k}^{-i}(t)\right) \dot{k}^j(t) \\ &+ e^{-\theta t} u_G\left(c^i(t), G(t)\right) \sum_{j \neq i} g^j_{k^s(t)}(\vec{k}(t)) \end{aligned} \tag{23}$$

Define

$$q^i(t) \;\equiv\; e^{\theta t} J_{k^i(t)}\big(t, k^i(t), \vec{k}^{-i}(t)\big) \tag{24}$$

$$\lambda^i_j(t) \;\equiv\; e^{\theta t} J_{k^j(t)}\big(t, k^i(t), \vec{k}^{-i}(t)\big) \tag{25}$$

$$\nu^i(t) \;\equiv\; e^{\theta t} \chi^i(t) \tag{26}$$

Equations (20) and (21) coincide with (5) and (6), respectively. Therefore, (9) and (10) hold also here (with the co-state defined as in (24) and the multiplier as in (26)).

To obtain the law of motion for the co-state in the feedback Nash equilibrium, we proceed as follows. Taking the time-derivative of (24) gives

$$\begin{aligned} \dot{q}^i(t) \;=\; & \theta e^{\theta t} J_{k^i(t)}\big(t, k^i(t), \vec{k}^{-i}(t)\big) \;+\; e^{\theta t} J_{t k^i(t)}\big(t, k^i(t), \vec{k}^{-i}(t)\big) \\ & + e^{\theta t} J_{k^i(t) k^i(t)}\big(t, k^i(t), \vec{k}^{-i}(t)\big) \dot{k}^i(t) \;+\; e^{\theta t} \sum_{j \neq i} J_{k^j(t) k^i(t)}\big(t, k^i(t), \vec{k}^{-i}(t)\big) \dot{k}^j(t) \end{aligned} \tag{27}$$

Using (21) and (24)-(27) in (22) gives[5]

$$\begin{aligned} \dot{q}^i(t) \;=\; & \Big[\theta - f_K(K(t),L) - f_{KK}(K(t),L)\big(k^i(t) - \bar{k}(t)\big) - \sum_{j \neq i} g^j_{k^i(t)}\Big] q^i(t) \\ & - \sum_{j \neq i} \lambda^i_j \Big[(k^j(t) - \bar{k}(t)) f_{KK}(K(t),L) - x^j_{k^i(t)} \Big] \;+\; \nu^i \sum_{j \neq i} g^j_{k^i(t)} \end{aligned} \tag{28}$$

Similarly, taking the time-derivative of (25), and combining with (23), we obtain

$$\begin{aligned} \dot{\lambda}^i_s(t) \;=\; & \big[\theta - f_K(K(t),L)\big] \lambda^i_s(t) \;-\; \sum_{j \neq i} \lambda^i_j \Big[f_{KK}(K(t),L)\big(k^j(t) - \bar{k}(t)\big) - x^j_{k^s(t)} \Big] \\ & - q^i(t)\, (k^j(t) - \bar{k}(t)) f_{KK}(K(t),L) \;-\; [q^i(t) - \nu^i(t)] \sum_{j \neq i} g^j_{k^s(t)} \end{aligned} \tag{29}$$

As in the open-loop equilibrium of section 3, individuals with lower capital (i.e. higher co-state q) would have a lower growth in her co-state [follows from equation (28)], implying that the co-states (and capital) converge over time, until all individuals own the average capital stock and all individuals are contributors. In steady state, therefore, equations (28) and (29) give $\lambda^i_s = \lambda = q$ for all i,s. Consequently

[5] Also using the constant returns-to-scale property, $f_{LK} = -f_{KK}K/L$.

$$\theta = f_K(K^*, L) - \sum_{j \neq i} c^j_{k^i}(\vec{k}^*) \tag{30}$$

The last term is the change in private consumption of all individuals except individual i, when individual *i*'s capital is increased. If other individuals' private consumption increase when one individual's capital is increased, then the right hand side of (30) is smaller than otherwise, implying that the steady state capital stock is smaller. The result depends on the individuals' conjectures of the reaction functions of the other individuals. For steady-state conjectures (i.e. reactions consistent with steady-state variations[6]), this is always the case [see the appendix].

Proposition 2. *In the feedback Nash equilibrium the following holds*:

1. *In the long run (steady state) all individuals will individually contribute the same amount towards the public good, regardless the level of capital they started with.*
2. *If individuals apply steady-state conjectures, and if private consumption is normal (inferior) then the steady state capital stock is smaller (larger) than the one in the open-loop equilibrium.*
3. *Steady state public goods provision is not necessarily increasing in population size even if the public good is a normal good.*
4. *The steady state number of contributors increase one-for-one with population size.*

When consumption is normal, an increase in individual *i*'s capital stock will increase the private consumption of the other individuals (and lower their public goods provision). This implies that an individual has an incentive of lowering his own savings, in order to make the others to contribute. However, everyone is reasoning in the same way, implying a lower steady state capital stock. If private consumption is inferior, each individual has an incentive to save more (to induce the others of providing more of the public good). The result is a larger steady-state capital stock.

The reason for the third result is that the response of the individuals to an increase of the capital stock of another individual depends on the number of individuals. For steady-state conjectures that derivative is typically larger for economies with more individuals. This means that in larger economies, the incentive to strategically choose the capital stock becomes

[6] See Itaya and Shimomura (2001).

stronger. If consumption is normal, then everything else equal, individuals have an incentive to save less in a larger economy. This effect may even undo the effect in Proposition 1. An example where this can happen is when $f(K,L)=AK^{\alpha}L^{1-\alpha}$ and $u(c,G) = U(c) + V(G)$, and $U(.)$ and $V(.)$ are negative exponential. Then steady-state aggregate public goods provision, G^* is increasing in L as long as $L < (1-\alpha)/\alpha$ and decreasing in L for $L > (1-\alpha)/\alpha$ (see the Appendix).

We now turn to a comparison across commitment regimes.

Corollary. *If individuals apply steady-state conjectures, the following holds*:

1. *When both the private and the public goods are normal, the steady-state public goods provision is smaller in the feedback equilibrium than in the open-loop equilibrium.*
2. *When the private good is inferior, the steady-state public goods provision is greater in the feedback equilibrium than in the open-loop equilibrium.*

Thus, when all goods are normal, reducing the degree of commitment (moving from open-loop to Markov perfect) lowers the steady-state public goods provision. The reason is that normality of private consumption gives an incentive for individuals to lower their savings. The lower steady state capital stock implies a lower public goods provision in the aggregate, if the public good is normal.

When private consumption is inferior, on the other hand, each individual has an incentive to save more, in turn implying a larger steady-state capital stock. If private consumption is inferior, then the public good is necessarily normal, and therefore a larger capital stock increases the steady-state level of the public good.

5. CONCLUSIONS

In this paper we have analysed voluntary public goods provision in a dynamic economy, where individuals can accumulate capital. The purpose was to solve for the equilibrium under two different assumptions on commitment: full commitment (open-loop Nash equilibrium) and no commitment (feedback Nash equilibrium).

In a dynamic setting, not only private provision public goods is chosen strategically, but also capital accumulation. An individual with lower initial capital stock, will find that by increasing her savings on the margin will lower the returns to capital and increase the returns to labour. Since the individual owns relatively more labour than capital, an individual with a small capital endowment has an incentive of saving relatively more than richer individuals. This will make individuals' savings converge to equality, in turn making all individuals contributing to the public good.

In the feedback Nash equilibrium, each individual can affect the contributions of others by strategically varying their capital stock. If consumption is normal (and individuals apply steady-state conjectures) then an increase in one individual's capital stock will cause the other individuals to provide less of the public good (and consume more of the private). This gives each individual an incentive to save less than otherwise. This incentive is not present when individuals can commit to their future provision (because then the future capital stock cannot affect future behaviour). The result is a lower steady state capital stock in the feedback Nash equilibrium (greater if private consumption is inferior).

Next we compared steady states by varying the population size. In contrast to a static economy, in larger societies there are more contributors. In the open-loop Nash equilibrium, if (and only if) private consumption is normal, each individual contributes less in large societies. Furthermore, if (and only if) public consumption is normal, larger societies consume more of the public good, however it is converging to a finite limit [like in Andreoni (1988)]. This is not necessarily true in the feedback Nash equilibrium. We showed an example there total public goods provision was increasing in population size for small populations and decreasing for large populations.

Finally we compared public goods provision across degrees of commitment. If both private and public consumption are normal, then the steady state public provision is greater in the open-loop Nash equilibrium than in the feedback Nash equilibrium. If private consumption is inferior, the opposite is true.

Our model is applicable to strategic interaction among countries. If at least one of the production factor is mobile, the capital poor countries would catch up with the rich, and in the long run they would too be voluntary contributors to a global public good.

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APPENDIX

Proof of Proposition 2

To solve for the steady-state conjectures we proceed as follows.

In steady state, for all individuals, the following holds:

$$u_c(c^i, G^{-i}+g^i) = u_G(c^i, G^{-i}+g^i) \tag{31}$$

$$f(K^{-i}+k^i, L) = \sum_{j\neq i} c^j + c^i + G^{-i} + g^i \tag{32}$$

Differentiating through w.r.t k^i gives

$$(u_{cc} - u_{Gc})\, c^i_{k^i} = (u_{GG} - u_{cG})\left(G^{-i}_{k^i} + g^i_{k^i}\right) \tag{33}$$

$$f_K = \sum_{j\neq i} c^j_{k^i} + c^i_{k^i} + G^{-i}_{k^i} + g^i_{k^i} \tag{34}$$

Use (30) in (34) to obtain

$$\theta = c^i_{k^i} + G^{-i}_{k^i} + g^i_{k^i} \tag{35}$$

Substitute (35) into (33)

$$c^i_{k^i} = \theta \frac{u_{GG} - u_{cG}}{u_{cc} + u_{GG} - 2u_{cG}} \tag{36}$$

and (36) into (35)

$$G^{-i}_{k^i} + g^i_{k^i} = \theta \frac{u_{cc} - u_{cG}}{u_{cc} + u_{GG} - 2u_{cG}} \tag{37}$$

Next, use equation (31) for individual *j* and differentiating through w.r.t k^i gives

$$(u_{cc} - u_{Gc})\, c^j_{k^i} = (u_{GG} - u_{cG})\left(G^{-i}_{k^i} + g^i_{k^i}\right) \tag{38}$$

Consequently

$$c^j_{k^i} = c^i_{k^i} \tag{39}$$

Then (39) and (37) together with (30) gives

$$\theta = f_K(K^*, L) - (L-1)\,\theta \frac{u_{GG} - u_{cG}}{u_{cc} + u_{GG} - 2u_{cG}} \tag{40}$$

The denominator in the extra term on the right is negative (by concavity of *u*). If u_{GG}-u_{cG} is negative (which is the condition for normality of *c*, since u_c=u_G), then the extra term is negative. Consequently f_K has to be larger for the right-hand side to equal the discount rate. this implies that the steady-state capital stock is lower. QED

To prove part 3, it is enough to provide an example. When $u(c,G) = v(c) + v(G)$, and $v(.)$ is negative exponential, then (36) becomes (by also using (39):

$$c_{k^i}^j \;=\; \theta \frac{v_{GG}}{v_{cc} + v_{GG}} \;=\; \theta \frac{v_{GG}/v_G}{v_{cc}/v_c \;+\; v_{GG}/v_G} \;=\; \frac{\theta}{2} \qquad (41)$$

Using (41) in (30) gives $\theta = f_K(K^*,L) - (L-1)\theta/2$, which in turn, by using $f(K,L)=AK^{\alpha}L^{1-\alpha}$ gives K^*/L as a function of L,

$$\frac{K^*}{L} \;=\; \left(\frac{\theta}{2}\frac{1+L}{\alpha A}\right)^{\frac{1}{\alpha-1}} \qquad (42)$$

Since the utility function is the same for both private and public consumption, marginal utilities are equalised when $c = G$. Then (11) implies $f(K^*,L)/L = (1+L)G/L$. Using the Cobb-Douglas production technology, and (42), gives

$$A\left(\frac{\theta}{2}\frac{1+L}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}} \;=\; \frac{1+L}{L}G^* \qquad (43)$$

Differentiating gives

$$\frac{\partial G^*}{\partial L} \;=\; \frac{1-\alpha-\alpha L}{L(1+L)(1-\alpha)}G^* \qquad (44)$$

Thus, G^* is increasing in L as long as $L < (1-\alpha)/\alpha$ and decreasing in L for $L > (1-\alpha)/\alpha$.

QED