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OPTIMAL DYNAMIC TAXATION WITH INDIVISIBLE LABOUR

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How should a government arrange taxes on labour and capital over time? To provide an answer, we develop the field of optimal dynamic taxation further by (i) incorporating indivisible labour and (ii) analysing the short-run dynamics of the capital and labour taxes under the second-best programme. We derive two classes of preferences for which the optimal capital tax reaches zero in a finite time. If leisure is normal, the labour tax is gradually increased for a period and then kept constant, and, if leisure is neutral, labour is not taxed at all. Finally, we analyse the dynamics of labour supply under the optimal tax programme.

1 Introduction

Taxation of factors of production and its macroeconomic effects are the subject of an ongoing debate in the policy arena. In the European Union, social security contributions, consumption and environmental taxation constitute important sources of government revenue, less so in the USA. Corporate and property taxes in Europe generate less revenue than in the USA and Japan. In 1998, personal income taxes accounted for 24 per cent of total tax revenue in the European Union and 41 per cent in the USA (Joumard, 2001).

Recently, the Commission of the European Union published a report arguing for a substitution away from labour taxation towards capital, consumption and environmental taxation (European Commission, 1996). The main objective of this proposed tax reform is to increase employment.

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Indeed, since the mid-1990s, marginal tax rates on labour income have been cut in several European Union countries (Austria, Germany, Ireland, the Netherlands, Spain, Sweden and the UK). In addition, a number of tax-credit schemes targeting married people, families with children and other typologies of individuals have substantially reduced the income tax burden on workers (Joumard, 2001).

The average tax wedge (which includes personal income taxes and social security contributions) has decreased between 1991 and 2000 in France, Italy, the Netherlands, Denmark, Norway, the UK and Ireland and outside Europe in the USA, New Zealand and Mexico (Joumard, 2001).

Nevertheless, according to the analysis by Gorter and de Mooij (2001), the capital income tax rates, including tax rates on interest, dividend and retained profits, for a sample of 15 European Union countries have also decreased between 1990 and 2000. In particular, the mean tax rate on interest has decreased from 46 to 37 per cent, on dividend from 55 to 52 per cent and on retained profits from 62 to 51 per cent. This trend is due to a decrease in corporate income and in personal income tax rates. The reason for the decrease can be found in a greater mobility of capital in a global economy and in the consequent enhanced international tax competition in capital taxation.

In Europe, increases have been recently registered in the VAT rates and environmental taxes, an example being the introduction in the UK of the Climate Change Levy in 2001 (Joumard, 2001; Martinez-Serrano and Patterson, 2003).

In the USA, the Economic Growth and Tax Relief Reconciliation Act of 2001 includes a reduction in income tax rates, beginning in 2001, in order to foster economic growth. The pre-2001 marginal income tax rates of 15, 28, 31, 36 and 39.6 per cent have been replaced with a more streamlined rate structure of 10, 15, 25 and 33 per cent. No provision has been considered for the capital tax. However, a new proposal of reforms, announced at the beginning of 2003, contemplates the abolition of tax on stock dividends (Martinez-Serrano and Patterson, 2003).

Answer to the debate on the tax mix lies in the theory of optimal dynamic taxation. This theory is mainly concerned with the following question: how should a government arrange taxes over time? A central result is that a capital tax should be zero in a steady state (Judd, 1985; Chamley, 1986). This is a robust second-best result,[1] and is verified under various frameworks.[2]

[1]There are two ways of introducing the second-best. The first is when the government has to raise an exogenously specified amount of revenue without recourse to lump-sum taxation. The second-best tax system then minimizes the distortions. The second alternative is to introduce heterogeneous individuals. The government then resorts to distortionary taxation for redistributive reasons. Chamley (1986) takes the first view, and Judd (1985) the second.

[2]See Atkeson *et al.* (1999) and Renström (1999) for surveys.

However, there is little analysis of the optimal taxes out of a steady state. An exception is Chamley (1986) who shows that the capital tax reaches zero in finite time for an example where the utility function is iso-elastic in consumption. Jones *et al*. (1993) have computed transition paths of taxes numerically.[3] Also, little attention has been given to the labour tax. There is in principle nothing that precludes the government from taxing capital and labour at the beginning of the optimization period, accumulating assets, and levying no taxes in the steady state.[4]

The purpose of this paper is to analyse optimal tax policy and its macroeconomic impact in an optimal dynamic taxation model with indivisible labour. A central result is the shift of the burden of taxation from capital to labour. We also find conditions under which employment increases after the tax reform.

In this paper we take Chamley's (1986) analysis further in two main directions. First we explore the dynamics of the tax paths, particularly whether capital income tax goes to zero in finite or infinite time. The answer to this question depends on the preference structure. We derive two classes of preferences for which the optimal capital tax approaches zero in a finite time (these are necessary and sufficient conditions). Second, we explore the optimal labour tax implications of the Chamley model. What preference structures will leave labour untaxed at all times? The issue is important because one may question whether the labour income tax is also zero in the steady state like capital tax. If the government can accumulate capital, it could raise all necessary revenues by taxing capital and labour at the beginning of the optimization period, and set all taxes to zero at the steady state.

Both these questions are answered in a framework with indivisible labour supply, as in Hansen (1985) and Rogerson (1988). We choose a model with indivisible labour for two reasons. The first reason is macroeconomic realism. An indivisible-labour economy explains the business cycle stylized facts better than a divisible-labour model. In particular, the volatility of employment is better taken care of in a model with indivisible labour (see Hansen, 1985) than divisible labour.[5]

The second reason why we use a model with indivisible labour is

[3]Jones *et al*. (1993) address the issue of optimal labour taxation including physical and human capital. However, their analysis is mostly based on simulation with specific functional forms, and does not admit a closed form solution with a fairly general preference structure.

[4]Jones *et al*. (1997) address the issue of optimal labour taxation including human capital in addition to physical capital, so that the labour tax has an intertemporal distortion. They show that there are certain cases when the labour tax is zero in steady state. See also Reinhorn (2003) for a clarification of those results, in particular regarding interior solutions of a model with human capital in addition to physical capital.

[5]Greenwood and Huffman (1996) find additional implications for the natural rate of unemployment using an indivisible-labour model. Mulligan (1999) finds that the optimal tax implications differ between indivisible-labour and divisible-labour models.

modelling convenience. With indivisible labour, we can establish a connection between the household's demand for unemployment insurance and the normality of leisure.[6] When leisure is normal, in order to raise the tax base, the fiscal authority would tax labour to induce the household to work harder. When labour supply is indivisible, and leisure is normal, the individual buys insurance to equate the utility gain from not working to the utility cost of the insurance purchase. The novelty of our approach is that we derive an exponential class of preferences with non-separable leisure for which this insurance demand is zero. This means leisure is a *neutral* good and the immediate implication is that labour should remain untaxed. In this scenario, labour should be untaxed at all times. For the same exponential preferences, capital should be taxed at 100 per cent for a finite time period, and then not taxed at all.

The paper is organized as follows. Section 2 outlines the economy. In Section 3 the key optimal tax results are derived. Section 4 shows the impact of the optimal tax programme on labour supply. Section 5 presents policy implications. Section 6 concludes.

2 The Economy

As in Hansen (1985) and Rogerson (1988), we consider an economy where labour supply is indivisible, so that individuals can work either full time, with h_0 being time spent working, or not at all. In each period the household member participates in an employment lottery, choosing the probability of working. This makes her wage income uncertain. It is assumed that households have access to an actuarially fair insurance market, where they can buy unemployment insurance. This makes the economy Pareto efficient in the absence of distortionary taxation. In this way we preserve the standard second-best framework: the underlying economy is Pareto efficient in the absence of government intervention and the government only has access to distortionary instruments, which are used so as to minimize these distortions.

2.1 Individual Economic Behaviour

In each period the household member participates in an employment lottery. With probability $\alpha(t)$, the individual works full time, h_0, and with probability $[1 - \alpha(t)]$ the individual is unemployed. She has access to an insurance market where she buys unemployment insurance, $y(t)$. The household's consumption ($c^s(t)$) and asset accumulation ($\dot{a}^s(t)$) are thus potentially contingent on whether the household works ($s = 1$) or not ($s = 2$). There is no intrinsic

[6]Basu and Renström (2002) analysed the sign of the optimal labour tax in an indivisible-labour economy in relation to the household's insurance demand, and established a connection between insurance demand and normality of leisure.

uncertainty, which means that preferences and technology are non-stochastic. The household thus solves the following maximization problem:

$$J(a_0) \equiv \max_{c,y,\alpha} \int_0^{\infty} \mathrm{e}^{-\theta t} \{\alpha(t)u[c^1(t), 1-h_0] + [1-\alpha(t)]u(c^2(t), 1)\}\mathrm{d}t \tag{1}$$

$$\dot{a}^1(t) = \rho(t)a(t) + \omega(t)h_0 - p(\alpha(t))y(t) - c^1(t) \tag{2}$$

$$\dot{a}^2(t) = \rho(t)a(t) + y(t) - p(\alpha(t))y(t) - c^2(t) \tag{3}$$

$$a(0) = a_0 \tag{4}$$

where $a(t)$ equals the sum of outstanding public debt, $b(t)$, and the capital stock, $k(t)$, that earns the after-tax interest at rate $\rho(t) = [1 - \tau^{\mathrm{K}}(t)]r(t)$, and $\omega(t) = [1 - \tau^{\mathrm{L}}(t)]w(t)$ is the after-tax wage; $r(t)$ and $w(t)$ are the rental and wage rates, respectively, $\tau^{\mathrm{K}}(t)$ and $\tau^{\mathrm{L}}(t)$ are the proportional tax rates on capital and labour income, respectively, and $p(\alpha(t))$ is the competitive price of insurance. The insurance company behaves competitively and maximizes the expected profit, $p(\alpha(t))y(t) - [1 - \alpha(t)]y(t)$, which gives rise to the zero-profit condition, $p(\alpha(t)) = 1 - \alpha(t)$.[7]

Substituting the zero-profit condition into (2) and (3), the current value Hamiltonian of the representative household can be written as

$$\begin{aligned} H = {} & \alpha(t)u(c^1(t), 1-h_0) + [1-\alpha(t)]u(c^2(t), 1) \\ & + \alpha(t)q^1(t)\{\rho(t)a(t) + \omega(t)h_0 - [1-\alpha(t)]y(t) - c^1(t)\} \\ & + [1-\alpha(t)]q^2(t)[\rho(t)a(t) + \alpha(t)y(t) - c^2(t)] \end{aligned} \tag{5}$$

The first-order conditions are (subscripts denoting partial derivatives):

$$\frac{\partial H}{\partial c^1(t)} = u_c(c^1(t), 1-h_0) - q^1(t) = 0 \tag{6}$$

$$\frac{\partial H}{\partial c^2(t)} = u_c(c^2(t), 1) - q^2(t) = 0 \tag{7}$$

$$\frac{\partial H}{\partial y(t)} = q^1(t) - q^2(t) = 0 \tag{8}$$

[7]The household randomizes the labour supply decision in this setting by choosing a probability of work $\alpha(t)$. A realistic description of this arrangement is that the representative household consists of a family of N members. In each period the household decides the proportion, $\alpha(t)$, of members working. The labour supply is then $\alpha(t)h_0N$. The household can buy insurance in a competitive market to diversify the income uncertainty arising from $(1 - \alpha(t))N$ of its members not working. After choosing the probability of work, $\alpha(t)$, the household is pre-committed to it, and cannot renege. The insurance company then charges the actuarially fair premium, $1 - \alpha$. This rules out adverse selection in the model. The household then realizes that, when choosing work probability, the insurance premium is a linear function of the probability of its working.

Using (6), (7), and (8) it follows that

$$u_c\left(c^1(t), 1-h_0\right) = u_c\left(c^2(t), 1\right) = q(t) \tag{9}$$

which is a result of perfect insurance. In other words, by buying insurance, the individual equalizes the marginal utilities across states. Equation (9) gives the optimal time paths of state-contingent consumption, $c^1(q(t))$ and $c^2(q(t))$, as functions of the co-state variable $q(t)$. However, this does not necessarily imply that the household will equalize consumption across states. For consumption equalization, one requires an additional restriction on the preferences that the utility function is additively separable between consumption and leisure, meaning $u_{c(1-h)} = 0$. It turns out that without any such restriction on the preferences, the household will not choose to have full consumption insurance as in Hansen (1985). This can be seen from (6), (7) and (8). Since $1 - h_0$ is not equal to unity, c^1 cannot equal c^2 unless $u_{c(1-h)}$ is equal to 0. Next, since $q^1(t) = q^2(t)$ it follows that the optimal asset-holding decisions must satisfy

$$\dot{a}^1(t) = \dot{a}^2(t) \tag{10a}$$

$$\dot{q}(t) = [\theta - \rho(t)]q(t) \tag{10b}$$

An individual's asset accumulation is thus independent of her employment history. This implies that individuals starting with the same a_0 will have the same $a(t)$ at all t, regardless of their employment history. Substituting (10a) in (2) and (3) gives

$$y(t) = \omega(t)h_0 + c^2(q(t)) - c^1(q(t)) \tag{11}$$

Notice now that the household chooses full insurance if the optimal consumption bundles are such that $c^1(q(t)) = c^2(q(t))$. In the absence of any restriction on $y(t)$, the household can choose to have positive, negative or zero insurance.[8] Finally, the optimal choice of $\alpha(t)$ must be such that

$$\frac{\partial H}{\partial \alpha(t)} = u\left(c^1(q(t)), 1-h_0\right) - u\left(c^2(q(t)), 1\right) - q(t)\left[c^1(q(t)) - c^2(q(t)) - \omega(t)h_0\right] = 0 \tag{12}$$

which upon the use of (11) can be rewritten as

$$u\left(c^2(q(t)), 1\right) - u\left(c^1(q(t)), 1-h_0\right) = q(t)y(t) \tag{12$'$}$$

The household chooses to buy a positive insurance, $y(t)$, if the utility gain from not working balances the utility cost of the insurance purchase.

[8]One needs to be careful about the non-negativity constraint on consumption while thinking about negative unemployment benefit. $y(t)$ can be negative as long as $c^2(t)$ is non-negative. We assume interior solutions, meaning $c^2(t) > 0$.

2.2 Production

There is large number of competitive firms in the economy each operating under the following constant returns to scale technology:

$$f(k(t), \alpha(t)h_0) = f_1 \cdot k(t) + f_2 \cdot \alpha(t)h_0 \tag{13}$$

2.3 The Government

The government taxes labour and capital income to finance an exogenously specified sequence of public spending, $g(t)$, the use of which is not explicitly modelled. It adjusts two tax rates, $\tau^L(t)$ and $\tau^K(t)$, continuously. The government is assumed to borrow and lend freely at the market rate of interest, $r(t)$. The government's budget constraint is therefore given by

$$\dot{b} = r(t)b(t) - \tau^K(t)r(t)a(t) - \tau^L(t)w(t)\alpha(t)h_0 + g(t) \tag{14}$$

with $b(0) = b_0$.

2.4 Equilibrium

The equilibrium is characterized by the following conditions:

(a) Facing $w(t)$, $r(t)$, $\tau^L(t)$, $\tau^K(t)$, the household chooses optimal sequences of $c(t)$, $a(t)$, $\alpha(t)$, $y(t)$ that solve the problem stated in (1), (2) and (3).[9]

(b) Given an exogenous stream of government spending, $g(t)$, the government pre-commits to a tax sequence, $\tau^L(t)$ and $\tau^K(t)$, and a debt sequence, $b(t)$, that satisfies the government budget constraint (14).

(c) Goods, labour and rental markets clear, meaning

$$\dot{k}(t) = f(k(t), \alpha(t)h_0) - \alpha(t)c^1(t) - [1-\alpha(t)]c^2(t) - g(t) \tag{15}$$

$$\omega(t) = [1-\tau^L(t)]f_2(k(t), \alpha(t)h_0) \tag{16}$$

$$\rho(t) = [1-\tau^K(t)]f_1(k(t), \alpha(t)h_0) \tag{17}$$

Notice that the equilibrium level of employment, $h(t)$ ($\equiv \alpha(t)h_0$) is determined by the time path of the probability of work, $\alpha(t)$. The equilibrium time path of $\alpha(t)$ can be determined in two steps. First, using (12) one determines the market clearing after-tax wage $\omega(t)$ as a function of $q(t)$. Define that equilibrium wage function as

$$\omega(t) = \Omega(q(t)) \tag{18}$$

Next, using (16) and (18), one can characterize the path of $\alpha(t)$ as a function of $k(t)$, $q(t)$ and $\tau^L(t)$ as follows:

$$\alpha(t) = \alpha(k(t), \tau^L(t), q(t)) \tag{19}$$

[9]We assume no-Ponzi games.

3 Optimal Dynamic Taxation

3.1 Government's Problem

We now solve for the optimal tax problem for the government for this economy with indivisible labour. The government solves a Ramsey problem for pre-committed tax sequences, $\tau^{L}(t)$ and $\tau^{K}(t)$, that maximize the household's utility functional (1) subject to its own budget constraint (14), the economy-wide resource constraint (15), the first-order optimality conditions (9), (10b) and (12), and a no-confiscation constraint on capital income as follows:[10]

$$\rho(t) \geq 0 \tag{20}$$

Using (16) and (17) and the constant returns to scale property of the production function, the government's budget constraint, (14), can be rewritten as

$$\dot{b}(t) = \rho(t)b(t) + \rho(t)k(t) + \alpha(t)\omega(t)h_0 - f(k(t), \alpha(t)h_0) + g(t) \tag{21}$$

We may write the government's current value Hamiltonian as follows (ignoring the time indices from now on):

$$\begin{aligned} H^{g} = {} & \alpha u(c^1, 1-h_0) + (1-\alpha)u(c^2, 1) + \mu[\rho b + \rho k + \alpha\omega h_0 - f(k, \alpha h_0) + g] \\ & + \lambda[f(k, \alpha h_0) - \alpha c^1 - (1-\alpha)c^2 - g] + \psi(\theta - \rho)q + v\rho \end{aligned} \tag{22}$$

In principle, the government faces the states k, b and q, and chooses the controls ρ and τ^{L}. For algebraic convenience, we pose the government's problem as follows. The government chooses the controls ρ and α. Then using the equilibrium sequence of α as in (19), one can determine the optimal labour tax, τ^{L}.[11] Denoting $u^1 = u(c^1, 1 - h_0)$ and $u^2 = u(c^2, 1)$, the first-order conditions facing the government are as follows:

$$\frac{\partial H^{g}}{\partial \alpha} = u^1 - u^2 + \mu(\omega h_0 - f_2 h_0) + \lambda(f_2 h_0 + c^2 - c^1) = 0 \tag{23}$$

$$\begin{aligned} & \frac{\partial H^{g}}{\partial q} = -\dot{\psi} + \theta\psi \\ & \Rightarrow \dot{\psi} = \rho\psi - \alpha u_c^1 c_q^1 - (1-\alpha)u_c^2 c_q^2 - \mu\alpha\Omega'(q)h_0 + \lambda[\alpha c_q^1 + (1-\alpha)c_q^2] \end{aligned} \tag{24}$$

[10] No such confiscation constraint is relevant for labour income taxation because, if labour income is confiscated by the government, it is optimal for the household to set $\alpha(t) = 0$ which means no production. On the other hand, in principle, the capital income can be confiscated and the government can eventually own all the capital to run production.

[11] It is straightforward to verify that, for given k and q, $\alpha'(\tau^{L}) < 0$ and hence $\alpha(\cdot)$ can be inverted with respect to τ^{L}.

$$\frac{\partial H^g}{\partial \rho} = \mu(b+k) - \psi q + v = 0 \tag{25}$$

$$\frac{\partial H^g}{\partial k} = \mu(\rho - f_1) + \lambda f_1 = \theta\lambda - \dot{\lambda} \tag{26}$$

$$\frac{\partial H^g}{\partial b} = \mu(\rho - \theta) = -\dot{\mu} \tag{27}$$

3.2 Preliminaries

The multiplier v in equation (25) is of importance in proving our results. As Chamley (1986) notices, $v(0) > 0$, meaning that the no-confiscation constraint binds at date zero. This can be seen from (25), since the multiplier associated with public debt is negative and the multiplier associated with the jump variable $q(0)$ is zero. A lot of insight regarding the time path can be gained by studying the law of motion for v. We know that v has to reach zero in finite time (to avoid marginal utility of consumption, q, going to infinity).

We may write v as a system of differential equations (see Appendix A):

$$\dot{v} = \theta v + Z \tag{28}$$

$$\dot{Z} = (\theta - \rho)Z + (\rho - f_1)(\lambda - \mu)S + M \tag{29}$$

where

$$Z \equiv (\lambda - q)S + \mu c^2 \tag{30}$$

$$S \equiv \alpha c_q^1 q + (1-\alpha)c_q^2 q \tag{31}$$

$$M \equiv \mu c_q^2 \dot{q} + (\lambda - q)\dot{S} \tag{32}$$

At some point in time, the no-confiscation constraint ceases to bind, say at t_1. Then, for $t \geq t_1$,

$$v(t) = \dot{v}(t) = 0$$

Next, (28) implies $Z(t_1) = 0$, and consequently $\dot{Z}(t) = 0$ for $t \geq t_1$. From (29) we can then see when $\rho(t) = f_1(t)$, for $t < \infty$ (i.e. when capital is untaxed in finite time). This happens when $M(t) = 0$, at least for $t \geq t_1$. We shall explore the conditions on preferences for this to be the case.

Notice that $Z(t) = 0$ in (30) helps signing a multiplier combination from date t_1 and onwards

$$\frac{q - \lambda}{-\mu} = \frac{c^2}{-S} \tag{33}$$

Thus, private marginal utility of assets exceeds public marginal utility of capital. This is helpful in exploring the labour tax later on.

Taking the time derivative of (33) gives

$$\dot{S} = \frac{\dot{q}}{q}\left[S + \alpha c^1_{qq}(q)^2 + (1-\alpha)c^2_{qq}(q)^2\right] + \dot{\alpha}\left(c^1_q q - c^2_q q\right) \tag{34}$$

Consequently

$$M = \frac{\dot{q}}{q}\left\{\mu c^2_q q + (\lambda - q)\left[S + \alpha c^1_{qq}(q)^2 + (1-\alpha)c^2_{qq}(q)^2\right]\right\} + \dot{\alpha}(\lambda - q)\left(c^1_q q - c^2_q q\right) \tag{35}$$

Since the time derivative of α is a function of technology (as well as preferences), a certain path of α that makes M zero at all dates cannot be guaranteed by restricting to a particular class of preferences. Therefore necessary and sufficient for $M = 0$ at all dates is that the terms in braces sum to zero and either

$$c^1_q q = c^2_q q \tag{36}$$

or α is constant. However, it turns out that preferences guaranteeing a constant α are inconsistent with making the terms within the braces sum to zero. Therefore, only condition (36) is of interest.

3.3 Optimal Tax Results

Proposition 1: The optimal capital income tax reaches zero in finite time if, and only if, either (i) or (ii) holds:

(i) the utility function is of the class

$$u(c, 1-h) = D + \frac{c^{1-\pi}}{1-\pi} + \phi(1-h_0) \tag{37}$$

(ii) the economy reaches a steady state in finite time.

Proof: If (36) holds, S, as defined in (31), reduces to

$$S = c^2_q q \tag{38}$$

Taking the time derivative of (38) and combining with (32) gives

$$M = \dot{q}\left[\mu c^2_q + (\lambda - q)\left(c^2_q + c^2_{qq} q\right)\right] \tag{39}$$

Using (33) gives

$$M = \mu c^2 \frac{\dot{q}}{q}\left[\frac{c^2_q q}{c^2} - \frac{c^2_{qq} q}{c^2_q} - 1\right] \tag{40}$$

Setting the bracketed term to zero and integrating gives (37) (see Appendix B). ■

The preferences in (37) imply full consumption insurance, i.e. consumption is equalized across states (this is because it is additively separable in consumption and leisure, implying that marginal utility of consumption can only be equalized across states if consumption itself is equalized across states). We will next look at the labour tax implied by those preferences.

Proposition 2: If preferences are of the class (37), then the labour tax is positive (at least from the date at which the non-confiscation constraint does not bind) and is constant over time from the date at which the non-confiscation constraint ceases to bind.

Proof: Plugging (12′) into (23) and using (11), one obtains

$$f_2 - \omega = \frac{(q-\lambda)y}{(\lambda-\mu)h_0} \tag{41}$$

Full insurance (implied by (37)) in (11) gives $y = \omega h_0$, $f_2 = \omega$, which means $\tau^{\mathrm{L}} = 0$. Then (41) becomes

$$\frac{\tau^{\mathrm{L}}}{1-\tau^{\mathrm{L}}} = \frac{q-\lambda}{\lambda-\mu} \tag{42}$$

Equation (31) proves that the right-hand side of (42) is positive. Taking the time derivative of (42) using (10b), (26) and (27), one obtains

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\tau^{\mathrm{L}}}{1-\tau^{\mathrm{L}}} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{q-\lambda}{\lambda-\mu} = (f_1-\rho)\frac{q-\mu}{\lambda-\mu} \tag{43}$$

Appendix C outlines the steps in deriving the second equality. Proposition 1 proved that the capital income tax is zero from date t_0 and onwards; consequently the right-hand side of (43) is zero. ■

We will next explore the possibility of the economy reaching a steady state in finite time. We then have to derive preferences that make real variables (prices and quantities) constant in finite time. For example, it is necessary that the after-tax wage becomes constant in finite time. However, from equation (12) we see that if preferences guarantee a constant after-tax wage, then it has to be constant at all dates. We find those preferences in the next lemma.

Lemma 1: The after-tax wage is constant if, and only if, the utility function is of the following exponential class:

$$u(c, 1-h) = D - \phi^h \mathrm{e}^{-Ac} \tag{44}$$

where D and A are constants ($A > 0$), and $\phi^h = \phi(h)$, with $\phi'(h) > 0$ and $\phi''(h) > 0$.

Proof: Taking the time derivative of (12) and setting it to zero gives

$$-\dot{q}(c^1 - c^2 - \omega h_0) + q\dot{\omega}h_0 = 0 \tag{45}$$

Thus, only $c^1 - c^2 - \omega h_0 = 0$ (i.e. zero insurance demand) guarantees a constant after-tax wage. Then (12) implies $u^1 = u^2$, i.e. utilities are equalized across states. Together with marginal utility equalization, we have $u^1/u^1_c = u^2/u^2_c$. For this to hold at all dates, the utility–marginal utility ratio has to be equal to a constant, say $-1/A$.

Integrating $u^1/u^1_c = -1/A$ gives (44) (see Appendix B). ■

Proposition 3: If preferences are of the class (44), then the labour tax is zero at all dates.

Proof: This follows by setting $y = 0$ in (41). ■

Corollary 1: If preferences are of the class (44), then capital's and labour's marginal products are constant at all dates.

Proof: By Lemma 1 the after-tax wage is constant. Since labour income is untaxed (by Proposition 3) the pre-tax wage (labour's marginal product) is also constant, as is capital's marginal product (by constant returns to scale). ■

Proposition 4: If preferences are of the class (44), then the economy reaches a steady state in finite time, say at t_1. Capital is taxed at 100 per cent up until date t_1, and is untaxed thereafter.

Proof: For preferences of the class in (44), S as defined in (31) is constant, and in particular $S = -1/A$. Then M as defined in (32) becomes (note that $c^2_q q = q/u^2_{cc} = u^2_c/u^2_{cc} = -1/A$)

$$M = \mu \frac{\rho - \theta}{A} \tag{46}$$

From time $t \geq t_1$ we have $Z(t) = \dot{Z}(t) = 0$; then (29) becomes (note that $S = -1/A$)

$$(\rho - f_1)(\lambda - \mu) = \mu(\rho - \theta) \tag{47}$$

where (46) has been used. Obviously, a steady state ($\rho = \theta$) implies that capital is untaxed, see (47). But if capital is untaxed and there is a steady state we must have $f_1 = \theta$. Since f_1 is constant (Corollary 1) already at t_1 it must equal θ at all times. Then (47) implies that capital is untaxed, and the economy is at a steady state, from time t_1 and onwards. This implies that asymptotic convergence to a steady state does not happen under the second-best tax programme. The only possibility left is that a steady state is not reached and that $f_1 \neq \theta$. The remainder of the proof investigates that.

Equation (33) gives $\lambda - \mu = q - \mu + \mu A c^2$, which substituted into (47) and using the definition of ρ gives

$$-\tau^{\mathrm{K}} f_1\left(\frac{q}{\mu} - 1 + Ac^2\right) = f_1 - \theta - f_1\tau^{\mathrm{K}} \tag{48}$$

Taking the time derivative of (48) (f_1 is constant by Corollary 1, q/μ is also constant) and using (48) gives

$$\dot{\tau}^{\mathrm{K}} = (\tau^{\mathrm{K}})^2 f_1\left(1 - \frac{f_1}{f_1 - \theta}\tau^{\mathrm{K}}\right) \tag{49}$$

This differential equation has two steady states: $\tau^{\mathrm{K}*} = \{0, 1 - \theta/f_1\}$. If $f_1 > \theta$, then regardless of the level of τ^{K} the capital tax converges to $1 - \theta/f_1$. This means that $\rho = f_1(1 - \tau^{\mathrm{K}}) = f_1\theta/f_1 = \theta$. However, this would imply convergence to a steady state, and thus contradicts the first part of the proof (asymptotic convergence does not happen). If $f_1 < \theta$, the level of τ^{K} must fall below $1 - \theta/f_1$, otherwise the tax rate would explode according to (49). For levels below $1 - \theta/f_1$ all τ^{K} go to zero asymptotically. However, if τ^{K} goes to zero asymptotically, then (47) would imply an asymptotic convergence to a steady state. Again, this is a contradiction. ■

4 Labour Supply Responses

In the policy debate, labour supply responses have been central in advocating different tax policies. In this section we explore whether there is any connection with labour supply responses and optimal policy. In particular, is it the case that optimal tax policy increases labour supply? As we shall see, there is little connection between optimal policy and labour supply responses. We shall concentrate on a special case where results can be derived. Our starting point is the preference structure in (37), for which the optimal policy was stated in Propositions 1 and 2. We shall make two further assumptions.

(a) Government expenditure gives utility to the individual. In particular, we replace equation (1) by

$$\begin{aligned} J(a_0) \equiv \max_{c,y,\alpha} \int_0^\infty \mathrm{e}^{-\theta t}\Bigg\{\alpha(t)\left[\frac{c^1(t)^{1-\pi}}{1-\pi} + \phi(1-h_0)\right] \\ + [1-\alpha(t)]\left[\frac{c^2(t)^{1-\pi}}{1-\pi} + \phi(1)\right] + \varepsilon\frac{g(t)^{1-\pi}}{1-\pi}\Bigg\}\mathrm{d}t \end{aligned} \tag{1$'$}$$

where $g(t)$ can be interpreted as a public good.[12]

(b) We assune Cobb–Douglas production. That is, equation (13) takes the form

$$f(k(t), \alpha(t)h_0) = Ak(t)^{\eta}[\alpha(t)h_0]^{1-\eta} \tag{13'}$$

The results would potentially depend on the time path of $g(t)$ (as well as its present value sum). We should concentrate on the most 'natural' time path of $g(t)$, which should be a derived time path. That the utility function of $g(t)$ is of the same form as of $c(t)$ helps in reducing the dynamic system of the economy under the optimal tax programme to a two-by-two system of differential equations.[13]

The Cobb–Douglas production function gives constant factor shares, again reducing the system of differential equations to two. In the two-by-two system we can do global analysis, which we could not have done for a larger system. This is important, since we want to focus on the out of steady state dynamics.

The first-order conditions to the government's problem with respect to after-tax prices and states remain unchanged. In addition we have the first-order condition with respect to $g(t)$:

$$\frac{\partial H^g}{\partial g} = \varepsilon g^{-\pi} + \mu - \lambda = 0 \tag{50}$$

Additive separability in consumption across states gives $c_1 = c_2$. Then (12) gives

$$q(t)\omega(t)h_0 = \phi(1) - \phi(1 - h_0) \tag{51}$$

Differentiating with respect to time and using (10b) gives

$$\dot{\omega}(t) = [\rho(t) - \theta]\omega(t) \tag{52}$$

Take the time derivative of (16), and use (13′) to obtain

$$\frac{\dot{\omega}(t)}{\omega(t)} = \frac{-\dot{\tau}(t)}{1-\tau(t)} + \eta\frac{\dot{k}(t)}{k(t)} - \eta\frac{\dot{\alpha}(t)}{\alpha(t)} \tag{53}$$

Next, using (43) and (42) and combining with (52) and (53) gives

$$\frac{\dot{\alpha}(t)}{\alpha(t)} = \frac{\dot{k}(t)}{k(t)} + \frac{\theta - r(t)}{\eta} \tag{54}$$

[12]Alternatively, we could have used $g(t)$ in production, as a public production factor. Similar results can be derived (available from the authors on request).

[13]Potentially the system of differential equations to an optimal tax problem can be large. For example, Chamley (1986) investigates local stability (still with iso-elastic utility). His system consists of five differential equations.

where $r(t) = f_1$, capital's marginal product. Equations (15) and (13′) give

$$\frac{\dot{k}(t)}{k(t)} = \frac{r(t)}{\eta} - \frac{c(t)+g(t)}{k(t)} \tag{55}$$

which combined with (54) yields

$$\frac{\dot{\alpha}(t)}{\alpha(t)} = \frac{\theta}{\eta} - \frac{c(t)+g(t)}{k(t)} \tag{56}$$

Equation (56) is the key in understanding the labour supply dynamics. The long-run (steady state) ratio between private plus public consumption and capital is θ/η. During the time period when capital is taxed at 100 per cent, private consumption is large initially and declines gradually. The consumption/capital ratio is then typically larger than its long-run value, implying by (56) that labour supply is decreasing over time. When capital ceases to be taxed, labour supply may either continue to decrease or increase. The exact condition is given in the next proposition.

Proposition 5: If preferences are of the class (1′), so that capital is untaxed after finite time, say from date t_1, then if the economy grows toward its steady state from date t_1, labour supply is increasing (decreasing) along the growth path if $\eta > \pi$ ($\eta < \pi$), and is constant if and only if $\eta = \pi$.

Proof: See Appendix D.

If capital's share, η, is large (relative to π), so capital is relatively more productive, the consumption trajectory is steeper than θ/η. That is, the household picks a lower consumption level at t_1, deferring consumption more along the growth path (with a low π the household is more willing to defer consumption). When $\eta = \pi$ the household consumes a fixed proportion of the capital stock at each instant of time, i.e. the trajectory is linear.

The key observation from this section is that optimal tax policy is independent of the dynamic labour supply response. The basic tax structure prescribed by Propositions 1 and 2 holds regardless.

5 Policy Implications

In this section we draw policy conclusions from the propositions stated in Sections 3 and 4. First, as a corollary of Proposition 1, a government should seek to reduce the capital tax toward zero, as an economy approaches a steady state. Obviously, if a steady state is reached in finite time (part (ii) of Proposition 1), then the capital tax should also be set to zero at a finite date. If the funding requirement is large, capital should be taxed for a longer period.

The intuition for the steady state result has to do with the optimality of uniform commodity taxation. A tax on capital is equivalent to taxing future consumption at a higher rate. If uniform commodity taxation is optimal then zero capital taxation is also optimal. In the static optimal tax literature (Atkinson and Stiglitz, 1972), conditions for uniform taxation have been derived. If utility is additively separable across commodities, then if the elasticities of the utility function are the same for all commodities, all commodities should carry the same tax rate. The utility function derived in Proposition 1 (equation (37)) is of this form. Therefore, the principle of taxing dated commodities at equal rates kicks in immediately (not just at the steady state). However, there is one difference from the static literature. It is optimal to tax away the capital already accumulated by the individual at the beginning of the optimal tax programme (as it is inelastic). This implies that capital is taxed at the maximum rate for a period of time, and then untaxed after this period.

Based on our model's predictions, we recommend that governments should set a time horizon for abolishing the capital tax. This time horizon should be longer the greater the funding requirement is. In this case, governments should not abolish the labour income tax, however. Instead, as stated in Proposition 2, the labour income tax should be increasing during the time period under which capital is taxed, and when capital ceases to be taxed, labour should be taxed at a constant rate. This in effect means a gradual substitution away from capital taxes to labour taxes, a proposal which is in contrast with the European Commission view (European Commission, 1996).

Furthermore, based on the results in Section 4, our policy recommendation is that tax policy should not be guided by employment considerations, which the European Commission is doing (European Commission, 1996). Our findings show that the optimal tax structure is independent of the labour supply response.

What happens to labour supply under the optimal tax programme is not clear. Typically labour supply would decrease during the first phase of the tax programme and then either decrease or increase toward its steady state level. The gradual reduction in labour at the beginning of the tax programme has not so much to do with the increased labour tax rate. It is rather due to the high capital tax, which induces the individual to de-cumulate capital, thereby lowering labour's marginal product. When capital is untaxed, and capital is accumulated again, this will increase labour's marginal product and may induce a gradual increase in labour supply. The central conclusion is that one cannot judge by looking at an economy's labour supply response whether the tax programme has been optimal, or even close to optimal. One can only do so by looking at the time paths of the tax rates themselves in conjunction with evidence on the preference structure.

Another important result is that if leisure is neutral (neither normal nor inferior), i.e. of the form (44), it is not optimal to tax labour at all (Proposition 3). Governments should then not tax labour, but only capital and only for a period. The reason capital is only taxed for a period is because a steady state is reached in finite time when leisure is neutral. Empirical investigation of eventual income effects on labour supply is important for prescribing optimal government policy.

In short, we have derived the following rules of thumb for policy-making purposes:

1. If the intertemporal elasticity of substitution is constant, one should tax capital for a period and then not at all. In addition, one should impose a labour income tax, which is increasing for a period and then constant forever.
2. If leisure is neutral, one should not tax labour at all. One should tax capital for a period and then not at all.
3. If leisure is normal and the intertemporal elasticity of substitution is not constant, one should lower the capital tax and raise the labour tax gradually.

6 Conclusion

How should a government arrange taxes on labour and capital over time? This is the central concern of this paper. The existing literature on this issue focuses more on the long-run implications of the tax policy, while our paper has a more short-run emphasis with connections to the actual tax policy of the European Union. As a vehicle of analysis, we extend the Chamley–Judd model to indivisible labour and examine the short-run time paths of the optimal labour and capital taxes. Whether it is optimal to tax labour and/or capital typically boils down to the form of the individual preferences. Under iso-elastic preferences, a central result of this paper is the shift of the tax burden away from capital towards labour. Another important finding is that, if leisure is neutral, labour should not be taxed at all and capital is taxed only for a finite period of time.

Furthermore, we find that the basic time structure of the optimal tax policy is independent of the dynamic labour supply response. Thus, the European Union's concern about the employment effect of taxes has to be carefully re-evaluated in the context of a dynamic model of optimal taxation. What is needed is a careful analysis of the preference structure while formulating a tax design. A future extension of this paper may be to evaluate the possible distortion/welfare cost of the actual tax policy implemented in many European Union countries using a benchmark model of the kind developed in this paper.

Appendix A: Derivation of (28)

Premultiplying (24) by q and exploiting the fact that $\Omega'(q) = (c^1 - c^2 - \omega h_0)/qh_0$, one obtains

$$q\dot{\psi} = q\rho\psi + (\lambda - q)[\alpha c_q^1 q + (1-\alpha)c_q^2 q] + \mu\alpha(\omega h_0 - c^1 + c^2) \tag{A1}$$

Next note that

$$\frac{\mathrm{d}}{\mathrm{d}t}(\psi q) = \dot{\psi}q + \psi\dot{q} = \dot{\psi}q + \psi(\theta - \rho)q \tag{A2}$$

Plugging (A2) into (A1) gives

$$\frac{\mathrm{d}}{\mathrm{d}t}(\psi q) = \theta\psi q + (\lambda - q)[\alpha c_q^1 q + (1-\alpha)c_q^2 q] + \mu\alpha(\omega h_0 - c^1 + c^2) \tag{A3}$$

Next note that

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mu a) = \dot{\mu}a + \mu\dot{a} \tag{A4}$$

Plugging (27) into (A4)

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mu a) = \mu(\theta - \rho)a + \mu\dot{a} \tag{A5}$$

Using (2), (3) and (10a), the household's budget constraint can be rewritten as

$$\dot{a} = \rho a + \alpha\omega h_0 - \alpha c^1 - (1-\alpha)c^2 \tag{A6}$$

which after plugging into (A5) gives

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mu a) = \theta\mu a + \mu[\alpha\omega h_0 - \alpha c^1 - (1-\alpha)c^2] \tag{A7}$$

Next noting that $a = b + k$, rewrite (25) as

$$v = \psi q - \mu a \tag{A8}$$

Taking the time derivative of (A8), one obtains

$$\dot{v} = \frac{\mathrm{d}}{\mathrm{d}t}(\psi q) - \frac{\mathrm{d}}{\mathrm{d}t}(\mu a) \tag{A9}$$

Using (A3) and (A7) in (A9) one obtains equation (28).

Appendix B: Derivation of Equation (44)

Inverting gives

$$u_c^s / u^s = -A \tag{B1}$$

for $s = \{1, 2\}$. Or equivalently

$$\frac{\mathrm{d}\ln(-u^s)}{\mathrm{d}c^s} = -A \tag{B2}$$

Integrating both sides with respect to c^s gives

$$\ln(-u^s) = N^s - Ac^s \tag{B3}$$

where N^s is any constant, possibly dependent on s.

Taking exponents of both sides gives

$$-u^s = B^s \mathrm{e}^{-Ac^s} \tag{B4}$$

where $B^s = \exp(N^s)$ and consequently is positive, and is the only term which can be a function of leisure, consequently yielding (44). ■

Appendix C: Derivation of Equation (43)

Note that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{q-\lambda}{\lambda-\mu}\right) = \left[\frac{(\mathrm{d}q/\mathrm{d}t)-(\mathrm{d}\lambda/\mathrm{d}t)}{\lambda-\mu}\right] - \left(\frac{q-\lambda}{\lambda-\mu}\right)\left[\frac{(\mathrm{d}\lambda/\mathrm{d}t)-(\mathrm{d}\mu/\mathrm{d}t)}{\lambda-\mu}\right] \tag{C1}$$

Next plug (10b), (26) and (27) into the right hand side of (C1) to obtain (43). ■

Appendix D

With preferences of the form (37), the consumption Euler equation (from date t_1 and onwards) is

$$\dot{c}(t) = [r(t)-\theta]c(t)/\pi \tag{D1}$$

Taking the time derivative of (50) and using (26) and (27) gives

$$\dot{g}(t) = [r(t)-\theta]g(t)/\pi \tag{D2}$$

Define

$$x(t) \equiv \frac{\dot{\alpha}(t)}{\alpha(t)} \tag{D3}$$

Taking the time derivative of (D3), using (56), (D1) and (D2) gives

$$\dot{x}(t) = \frac{c(t)+g(t)}{k(t)}\left[\frac{\dot{k}(t)}{k(t)} - \frac{r(t)-\theta}{\pi}\right] \tag{D4}$$

Use (55) to substitute for the time derivative of k, and (56) to substitute for $(c + g)/k$ (note the definition of x). Taking the time derivative of (D3), using (56), (D1) and (D2) gives

$$\dot{x}(t) = \left[\frac{\theta}{\eta} - x(t)\right]\left\{[r(t)-\theta]\frac{\pi-\eta}{\eta\pi} - x(t)\right\} \tag{D5}$$

Since r is capital's marginal product, take the derivative of (13′) with respect to k and then the time derivative, and use (54) then;

$$\frac{\dot{r}(t)}{r(t)} = \frac{1-\eta}{\eta}[\theta - r(t)] \tag{D6}$$

(D5) and (D6) form a system of autonomous differential equations in r and x. Under the assumption that the economy grows toward its steady state $r(t_1) > \theta$. If $\eta > \pi$ the stable trajectory requires $x(t_1) > 0$; then, by the definition of x, α is increasing. On the stable trajectory x gradually declines toward zero, but is positive along the transition. If $\eta < \pi$ the stable trajectory requires $x(t_1) < 0$, so α is decreasing. On the stable trajectory x gradually increases toward zero, but remains negative along the transition. Finally if $\eta = \pi$ the stable trajectory requires $x(t_1) = 0$ (r then gradually declines towards θ, and x remains at zero along the path).

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