APPENDIX A: Proof of Propositions 2 and 5

Proof of Proposition 2

Define the employment rate, ν, as

$$\nu \equiv \frac{N}{L} = \beta\left(1 - \frac{1}{\eta}\right)\frac{\hat{A}}{L}\left(\frac{\rho + \Theta - 1}{\Theta - 1}b\right)^{-\frac{1-\gamma}{1-\mu}}[e(1+\tau^e)]^{-\frac{\gamma}{1-\mu}}(1-\tau^w)^{\frac{1-\gamma}{1-\mu}} \tag{42}$$

where the right-hand side is equation (20), where we have substituted for w by using (13). Alternatively we may write (42) as

$$F \equiv \nu - (1-\tau^w)^{\frac{1-\gamma}{1-\mu}}(Sb)^{-\frac{1-\gamma}{1-\mu}} = 0 \tag{43}$$

where $S \equiv \left(\frac{\rho + \Theta - 1}{\Theta - 1}\right)[e(1+\tau^e)]^{\frac{\gamma}{1-\gamma}}\left(\beta\left(1-\frac{1}{\eta}\right)\frac{\hat{A}}{L}\right)^{-\frac{1-\mu}{1-\gamma}}$ (44)

Next, we shall rewrite the government's budget constraint, (22), by using the fact that government revenue, R, equals unemployment benefit plus public goods provision, i.e. $R = (L-N)b + X^g$. Then we have

$$\tau^w wN + \frac{\tau^e}{1+\tau^e}\frac{\gamma}{\beta}wN + \frac{\tau^e\sigma}{1+\tau^e}\left\{\frac{\rho}{\Theta-1}bN + Lb + wN\frac{1+\alpha(\eta-1)}{\beta(\eta-1)}\right\} = (L-N)b + X^g \tag{45}$$

Dividing by bL and premultiplying by $(1-\tau^w)$ and rearranging gives

$$\tau^w z\frac{N}{L} + \frac{N}{L}a = (1-\tau^w)\left[\frac{1+(1-\sigma)\tau^e}{1+\tau^e} + \frac{X^g}{bL}\right] \tag{46}$$

where

$$z \equiv \frac{\rho}{\Theta-1}\frac{1+(1-\sigma)\tau^e}{1+\tau^e} > 0 \tag{47}$$

and

$$a \equiv 1 + \frac{\tau^e\sigma}{1+\tau^e}\frac{\rho}{\Theta-1} + \frac{\tau^e}{1+\tau^e}\frac{\rho+\Theta-1}{\Theta-1}\left\{\frac{\gamma}{\beta} + \sigma\frac{1+\alpha(\eta-1)}{\beta(\eta-1)}\right\} > 1 \tag{48}$$

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Rewrite (46) in terms of the employment rate ν, then

$$G \equiv \nu - \frac{1-\tau^w}{a+z\tau^w}H = 0 \tag{49}$$

where

$$H \equiv \frac{1+(1-\sigma)\tau^e}{1+\tau^e} + \frac{X^g}{bL} \tag{50}$$

Equation (43) gives the employment rate ν as a function of the wage tax τ^w. It has the following properties as $\tau^w = \{0,1\}$

$$\nu(0) = (Sb)^{-\frac{1-\gamma}{1-\mu}}$$
$$\nu'(0) = -\frac{1-\gamma}{1-\mu}(Sb)^{-\frac{1-\gamma}{1-\mu}} \tag{51}$$

$$\nu(1) = 0$$
$$\nu'(1) = 0 \tag{52}$$

Similarly, equation (49) gives the employment rate as a function of the wage tax, with the following properties
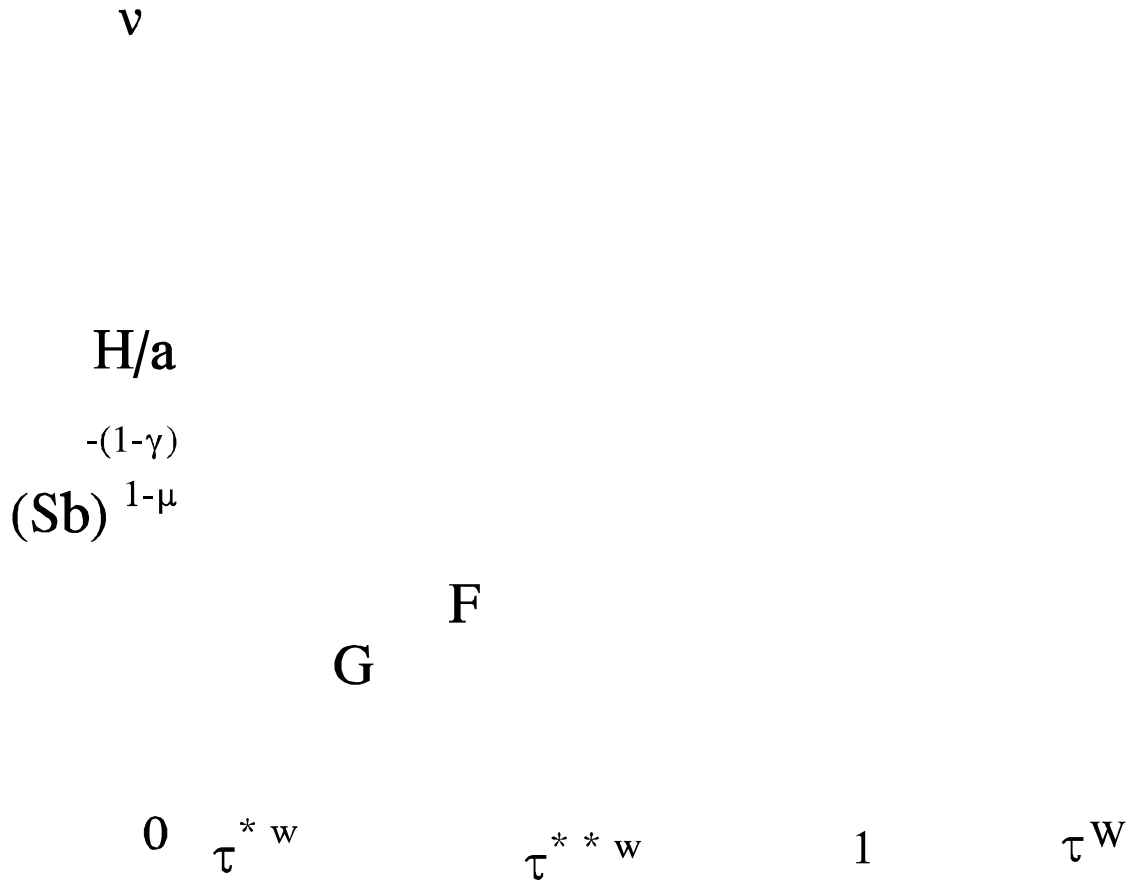
$$\nu(0) = H/a$$
$$\nu'(0) = -H(a+z)/a^2 \tag{53}$$

$$\nu(1) = 0$$
$$\nu'(1) = -H/(a+z) \tag{54}$$

Both function F and G decreasing and convex in τ^w, and take on value zero at $\tau^w=1$. Function F has zero slope at $\tau^w=1$, G not. If $H/a > (Sb)^{-(1-\gamma)/(1-\mu)}$ and F and G cross, they must cross exactly

twice, like in Figure 1.[1]

Figure 1: The General Equilibrium

ν

H/a

-(1-γ)

(Sb) $^{1-\mu}$

F

G

0   τ^{*w} τ^{**w} 1 τ^{W}

In this situation we have two equilibria, one Laffer efficient (τ^{*w}) and one Laffer inefficient (τ^{**w}). For the Laffer efficient equilibrium to be well defined we require that employment is less than 100% at this tax rate, i.e. $\nu(\tau^{*w})<1$. Sufficient for this to be true is that F intersects the vertical axis at $\nu \leq 1$, i.e. that $Sb \geq 1$. We require that G lies below F for values of the wage tax between the Laffer efficient level and the Laffer inefficient level. Sufficient for this being the case is that G lies below F at the Laffer maximum wage tax. The Laffer optimal wage tax is (see

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[1]If $H/a < (Sb)^{-(1-\gamma)/(1-\mu)}$ public expenditure is too small in relation to the government revenue from the energy tax, and there is no Laffer efficient wage tax. The wage tax has to be Laffer inefficient in this situation. If $H/a = (Sb)^{-(1-\gamma)/(1-\mu)}$ we have two equilibria. One in which the wage tax is zero, and one in which the wage tax is positive but Laffer inefficient. Both equilibria raise the same revenue. We shall not explore these cases further.

Lemma 1)

$$\bar{\tau}^{w} = \frac{1-\mu}{1-\gamma} \tag{55}$$

Function *F* at the Laffer optimal tax is

$$v = \left(\frac{\beta}{1-\gamma}\right)^{\frac{1-\gamma}{1-\mu}} (Sb)^{-\frac{1-\gamma}{1-\mu}} \tag{56}$$

Function *G* at the Laffer optimal tax is

$$v = \frac{\beta}{1-\gamma} H \Big/ \left(a + z\frac{1-\mu}{1-\gamma}\right) \tag{57}$$

Thus we need to show that

$$\frac{\dfrac{\beta}{1-\gamma}H}{a + z\dfrac{1-\mu}{1-\gamma}} \leq \left(\frac{\beta}{1-\gamma}\right)^{\frac{1-\gamma}{1-\mu}} (Sb)^{-\frac{1-\gamma}{1-\mu}} \tag{58}$$

or

$$\frac{H}{a + z\dfrac{1-\mu}{1-\gamma}} \leq \left(\frac{\beta}{1-\gamma}\right)^{\frac{\beta}{1-\mu}} (Sb)^{-\frac{1-\gamma}{1-\mu}} \tag{59}$$

First, $a+z(1-\mu)/(1-\gamma)$ is increasing in τ^{e}. Second H is decreasing in τ^{e}, and S is increasing in τ^{e}.

The larger τ^{e} is the more likely the condition is fulfilled.

For sufficiency it is thus enough to prove the condition at $\tau^{e}=0$. First we have

$$H|_{\tau^{e}=0} = 1 + \frac{\chi g}{Lb} \qquad (60) \qquad a + z\frac{1-\mu}{1-\gamma}\bigg|_{\tau^{e}=0} = 1 + \frac{1-\mu}{1-\gamma}\frac{\rho}{\Theta-1} \tag{61}$$

and

$$Sb|_{\tau^{e}=0} = \left(1+\frac{\rho}{\Theta-1}\right) e^{\frac{\gamma}{1-\gamma}} \left(\beta\left(1-\frac{1}{\eta}\right)\frac{\hat{A}}{L}\right)^{-\frac{1-\mu}{1-\gamma}} \tag{62}$$

Then (59) becomes

$$\frac{1+X^g/(Lb)}{1+\dfrac{1-\mu}{1-\gamma}\dfrac{\rho}{\Theta-1}} \leq \left(\frac{\beta}{1-\gamma}\right)^{\frac{\beta}{1-\mu}}\left(1+\frac{\rho}{\Theta-1}\right)^{-\frac{1-\gamma}{1-\mu}}e^{\frac{-\gamma}{1-\mu}}\beta\left(1-\frac{1}{\eta}\right)\frac{\hat{A}}{L} \tag{63}$$

Premultiply both sides by L and divide by $1-\gamma$ to obtain (23). QED

Proof of Proposition 5

Differentiating (34) with respect to τ^e, and premultiplying by $(1+\tau^e)^\sigma$ we have

$$(1+\tau^e)^\sigma\frac{\partial\tilde{W}}{\partial\tau^e} = -\frac{\sigma}{1+\tau^e}I+\frac{\partial I}{\partial\tau^w}\frac{\partial\tau^w}{\partial\tau^e}+\frac{\partial I}{\partial N}\frac{\partial N}{\partial\tau^e} \tag{64}$$

where $I\equiv(1-\tau^w)wN+(L-N)b+\Pi$. First we need to find the derivative of I with respect to τ^w, holding N constant. Since $\Pi=[1+(1-\mu)(\eta-1)]\beta^{-1}(\eta-1)^{-1}wN$ (which follows from (17) and (19)), we have

$$\left.\frac{\partial\Pi}{\partial\tau^w}\right|_N = \frac{\frac{1}{\eta}+(1-\mu)\left(1-\frac{1}{\eta}\right)}{\beta\left(1-\frac{1}{\eta}\right)}\frac{w}{1-\tau^w}N = \frac{\Pi}{1-\tau^w} \tag{65}$$

Then, since $(1-\tau^w)w$ is independent of τ^w (which follows from (13)), we have

$$\frac{\partial I}{\partial\tau^w} = \frac{\Pi}{1-\tau^w} \tag{66}$$

Next, since $(1-\tau^w)w$ is also independent of N we have

$$\frac{\partial I}{\partial N} = (1-\tau^w)w-b+\frac{\Pi}{N} \tag{67}$$

Substituting (66) and (67) into (64), and premultiplying by $(1-\tau^w)$ and by the determinant (28), gives

$$(1-\tau^w)\det\left.\frac{\partial\tilde{W}}{\partial\tau^e}\right|_{\tau^e=0} = -\sigma(1-\tau^w)I\det +\Pi\frac{\partial\tau^w}{\partial\tau^e}\det$$

$$+\left[(1-\tau^w)w-b+\frac{\Pi}{N}\right](1-\tau^w)\frac{\partial N}{\partial\tau^e}\det \tag{68}$$

Substituting for the derivatives by using (29) and (30), and for the determinant using (28) gives

$$(1-\tau^w)\det\frac{\partial\tilde{W}}{\partial\tau^e}\bigg|_{\tau^e=0} = -\sigma I w N\left(1-\frac{1-\gamma}{1-\mu}\tau^w\right)+\Pi\left[-e\bar{E}+eE^p\frac{\tau^w\beta}{1-\mu}\right]$$
$$+\left[(1-\tau^w)wN-bN+\Pi\right]\frac{(1-\gamma)e\bar{E}-\beta eE^p}{1-\mu}$$

(69)

First we have

$$(1-\tau^w)wN-bN = \left(\frac{\rho+\Theta-1}{\Theta-1}-1\right)bN = \frac{\rho}{\Theta-1}bN = \frac{\rho(1-\tau^w)}{\rho+\Theta-1}wN$$

(70)

where the first and last equality follows from (13). Next, substitute (70) into (69) and collect the terms involving $e\bar{E}$ and eE^p, then we have

$$(1-\tau^w)\det\frac{\partial\tilde{W}}{\partial\tau^e}\bigg|_{\tau^e=0} = -\sigma I w N\left(1-\frac{1-\gamma}{1-\mu}\tau^w\right)+\frac{e\bar{E}}{1-\mu}\left(\beta\Pi+(1-\gamma)\frac{\rho(1-\tau^w)}{\rho+\Theta-1}wN\right)$$
$$-eE^p\frac{\beta(1-\tau^w)}{1-\mu}\left[\Pi+\frac{\rho}{\rho+\Theta-1}wN\right]$$

(71)

Since $\Pi=wN/(\Theta-1)$ (follows from (17), (19) and (11)) we have

$$(1-\tau^w)\det\frac{\partial\tilde{W}}{\partial\tau^e}\bigg|_{\tau^e=0} = -\sigma I w N\left(1-\frac{1-\gamma}{1-\mu}\tau^w\right)+\frac{e\bar{E}wN}{1-\mu}\left(\frac{\beta}{\Theta-1}+(1-\gamma)\frac{\rho(1-\tau^w)}{\rho+\Theta-1}\right)$$
$$-\frac{eE^pwN}{1-\mu}\beta(1-\tau^w)\left[\frac{1}{\Theta-1}+\frac{\rho}{\rho+\Theta-1}\right]$$

(72)

Substituting for $e\bar{E}$ by using (34) and rearranging we have

$$(1-\tau^w)\det\frac{\partial\tilde{W}}{\partial\tau^e}\bigg|_{\tau^e=0} = -\sigma I w N\left(1-\frac{1-\gamma}{1-\mu}\tau^w\right)+\frac{\sigma LbwN}{1-\mu}\left(\frac{\beta}{\Theta-1}+(1-\gamma)\frac{\rho(1-\tau^w)}{\rho+\Theta-1}\right)$$
$$+\frac{eE^pwN}{1-\mu}\left[\frac{\beta}{\Theta-1}\left(\sigma\frac{s}{\gamma}+\tau^w\right)+\frac{\rho(1-\tau^w)}{\rho+\Theta-1}\left(1-\gamma-\beta+\sigma\frac{1-\gamma}{\gamma}s\right)\right]$$

(73)

Next, since by (6) $\sigma I=eE^h$, we have (by (31))

$$\sigma I = \sigma\frac{s}{\gamma}eE^p + \sigma Lb$$

(74)

substituting (74) into (73) gives

$$(1-\tau^w)\det\frac{\partial\tilde{W}}{\partial\tau^e}\bigg|_{\tau^e=0} = \frac{\sigma LbwN}{1-\mu}\left(\frac{\beta}{\Theta-1} + (1-\gamma)\frac{\rho(1-\tau^w)}{\rho+\Theta-1} - \left(1-\mu-(1-\gamma)\tau^w\right)\right)$$

$$+ \frac{eE^pwN}{1-\mu}\left[\left(\frac{\beta}{\Theta-1} - \left(1-\mu-(1-\gamma)\tau^w\right)\right)\sigma\frac{s}{\gamma} + \frac{\beta\tau^w}{\Theta-1}\right. \tag{75}$$

$$\left. + \frac{\rho(1-\tau^w)}{\rho+\Theta-1}\left(1-\gamma-\beta+\sigma\frac{1-\gamma}{\gamma}s\right)\right]$$

or by rearranging

$$(1-\tau^w)\det\frac{\partial\tilde{W}}{\partial\tau^e}\bigg|_{\tau^e=0} = \frac{\sigma LbwN}{1-\mu}\left(\frac{1}{\eta-1} + (1-\gamma)\frac{\rho(1-\tau^w)}{\rho+\Theta-1} + (1-\gamma)\tau^w\right)$$

$$+ \frac{eE^pwN}{1-\mu}\left[\left(\frac{1}{\eta-1} + (1-\gamma)\tau^w\right)\sigma\frac{s}{\gamma} + \frac{\beta}{\Theta-1}\tau^w\right. \tag{76}$$

$$\left. + \frac{\rho(1-\tau^w)}{\rho+\Theta-1}\left(1-\gamma-\beta+\sigma\frac{1-\gamma}{\gamma}s\right)\right]$$

Equation (76) is positive. QED

APPENDIX D: Solving for the economic equilibrium

Households

Maximising (1) s.t. (2) w.r.t. x_i gives the FOC

$$U_{x_i^h} = n^{\frac{1}{1-\eta}}\left(\sum_i (x_i^h)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1}(x_i^h)^{\frac{\eta-1}{\eta}-1} = \lambda p_i \tag{77}$$

where λ is the Lagrange multiplier. Dividing the FOC for x_i^h by the FOC for x_k^h gives

$$x_i^h = \left(\frac{p_k}{p_i}\right)^\eta x_k^h \tag{78}$$

Taking both sides to the power of $(\eta-1)/\eta$ and summing over i gives

$$\sum_i (x_i^h)^{\frac{\eta-1}{\eta}} = \sum_i p_i^{1-\eta} p_k^{\eta-1}(x_k^h)^{\frac{\eta-1}{\eta}} \tag{79}$$

Taking both sides to the power of $(\eta-1)/\eta$ and premultiplying by $n^{1/(1-\eta)}$ gives

The LHS of (92) is the composite commodity X^h, and part of the RHS is the price index P, thus

$$n^{\frac{1}{1-\eta}} \left(\sum_i (x_i^h)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} = n^{\frac{1}{1-\eta}} \left(\sum_i p_i^{1-\eta} \right)^{\frac{\eta}{\eta-1}} p_k^\eta x_k^h \qquad (80)$$

(92) is

$$X^h = n \, \mathbf{P}^{-\eta} \, p_k^\eta \, x_k^h \qquad (81)$$

Rearrange (93) to obtain the demand function

$$x_k^h = \frac{X^h}{n} \left(\frac{\mathbf{P}}{p_k} \right)^\eta \qquad (82)$$

Polluting good sector

Solving the minimisation problem in assumption A5 gives first-order conditions of the same form as (89), i.e.

$$n^{\frac{1}{1-\eta}} \left(\sum_i (x_i^e)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} (x_i^e)^{\frac{\eta-1}{\eta}-1} = \lambda p_i \qquad (83)$$

Following the same steps (90)-(94) gives the demand function for the polluting good sector, which is of the same form as the one of the households, i.e.

$$x_k^e = \frac{X^e}{n} \left(\frac{\mathbf{P}}{p_k} \right)^\eta \qquad (84)$$

Government

Solving the government's minimisation problem (assumption A6) gives first-order conditions of the same form as (89) and (94)

$$n^{\frac{1}{1-\eta}} \left(\sum_i (x_i^g)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}-1} (x_i^g)^{\frac{\eta-1}{\eta}-1} = \lambda p_i \qquad (85)$$

Following the same steps as of the household and polluting good sectors, gives the demand function for the government

$$x_k^g = \frac{X^g}{n}\left(\frac{P}{p_k}\right)^\eta \tag{86}$$

Firms

Firms minimise costs, subject to the level of y_j

$$C_j(y_j) = \min_{N_j, E_j^p} w N_j + \hat{e} E_j^p$$
$$\text{s.t. } \hat{A} N_j^\beta (E_j^p)^\gamma \geq y_j \tag{87}$$

where $\hat{e} \equiv (1+\tau^e)e$ and $\hat{A} \equiv AK^\alpha$. Equivalently we may substitute for N_j by using the constraint in (99), then we have

$$C_j(y_j) = \min_{E_j^p} w \hat{A}^{-\frac{1}{\beta}} y_j^{\frac{1}{\beta}} \left(E_j^p\right)^{-\frac{\gamma}{\beta}} + \hat{e} E_j^p \tag{88}$$

The FOC w.r.t. E_j^p is

$$-\frac{\gamma}{\beta} w \hat{A}^{-\frac{1}{\beta}} y_j^{\frac{1}{\beta}} \left(E_j^p\right)^{-\frac{\gamma}{\beta}-1} + \hat{e} = 0 \tag{89}$$

or equivalently

$$E_j^p = \gamma M \left(\frac{w}{\hat{e}}\right)^{\frac{\beta}{\mu}} y_j^{\frac{1}{\mu}} \tag{90}$$

where $\mu \equiv \beta + \gamma$ and

$$M \equiv \hat{A}^{-\frac{1}{\mu}} \left(\frac{\gamma}{\beta}\right)^{\frac{\beta}{\mu}} \frac{1}{\gamma} \tag{91}$$

Since the ratio of the marginal products is equal to the relative factor price

$$\frac{\beta}{\gamma} \frac{E_j^p}{N_j} = \frac{w}{\hat{e}} \tag{92}$$

we have

$$N_j = \beta M \left(\frac{w}{\hat{e}}\right)^{-\frac{\gamma}{\mu}} y_j^{\frac{1}{\mu}} \tag{93}$$

Substituting (102) and (105) into $C(.)$, gives the cost function

$$C(y_j) = M(\beta + \gamma) y_j^{\frac{1}{\mu}} w^{\frac{\beta}{\mu}} \hat{e}^{\frac{\gamma}{\mu}} \tag{94}$$

The firm's profit function is

$$\Pi_j = \max_{p_j} p_j(y_j) \, y_j - C(y_j) \tag{95}$$

Since households, the polluting good sector and the government have the same structure of their demand functions, each firm faces demand of the form

$$p_j(y_j) = \left(\frac{Y}{n y_j}\right)^{\frac{1}{\eta}} P \tag{96}$$

where in equilibrium $Y = X^h + X^e + X^g$. Each firm takes the aggregate production Y, and the price index P, as beyond its own control. Then the FOC w.r.t. y_i is

$$\left(1 - \frac{1}{\eta}\right) p_j(y_j) = C'(y_j) \tag{97}$$

At this stage we may look at the symmetric equilibrium and make the normalisation $P=1$. Substituting for the derivative of (106) gives

$$\left(1 - \frac{1}{\eta}\right) y_j^{-\frac{1}{\eta}} \left(\frac{Y}{n}\right)^{\frac{1}{\eta}} = M \, \hat{e}^{\frac{\gamma}{\mu}} \, w^{\frac{\beta}{\mu}} \, y_j^{\frac{1-\mu}{\mu}} \tag{98}$$

or rearranged

$$y_j^{\frac{1}{\mu}} = \left[\left(1 - \frac{1}{\eta}\right)\middle/ M\right]^{\frac{\eta}{\eta(1-\mu)+\mu}} \left(\frac{Y}{n}\right)^{\frac{1}{\eta(1-\mu)+\mu}} \hat{e}^{-\frac{\gamma}{\mu}\frac{\eta}{\eta(1-\mu)+\mu}} w^{-\frac{\beta}{\mu}\frac{\eta}{\eta(1-\mu)+\mu}} \tag{111}$$

Defining $\tilde{M} \equiv \left[\left(1 - \frac{1}{\eta}\right)\middle/ M\right]^{\frac{\mu(\eta-1)}{\eta(1-\mu)+\mu}}$ we get the firm's profit and factor demand equations

$$\Pi_j = \left[\frac{1}{\eta} + (1-\beta-\gamma)\left(1-\frac{1}{\eta}\right)\right] \tilde{M} \left(\frac{Y}{n}\right)^{\frac{\Theta+\Gamma}{\eta}} \hat{e}^{-\Gamma} w^{-\Theta+1} \tag{112}$$

$$N_j = \beta\left(1-\frac{1}{\eta}\right) \tilde{M} \left(\frac{Y}{n}\right)^{\frac{\Theta+\Gamma}{\eta}} \hat{e}^{-\Gamma} w^{-\Theta} \tag{113}$$

$$E_j^p = \gamma\left(1-\frac{1}{\eta}\right) \tilde{M} \left(\frac{Y}{n}\right)^{\frac{\Theta+\Gamma}{\eta}} \hat{e}^{-\Gamma-1} w^{-\Theta+1} \tag{114}$$

Next, defining $\tilde{A} \equiv \tilde{M} n^{\frac{(\eta-1)(1-\mu)}{1+(\eta-1)(1-\mu)}}$ substituting for $\tilde{M}$ in (112)-(114) and aggregating gives (8)-

(10).

Nash bargaining

$\Psi'(w)=0$ gives the global maximum, because Ψ is concave up to a point w^*, and $\Psi'(w)<0$ for $w>w^*$. We prove concavity by verifying that $\Psi''(w)<0$ for $w>w^*$.

First, for any function the following is true

$$\frac{d}{dw}\left(\frac{\Psi'(w)}{\Psi(w)}\right) = \frac{\Psi''(w)}{\Psi(w)} - \left(\frac{\Psi'(w)}{\Psi(w)}\right)^2 \tag{115}$$

or rearranged

$$\frac{\Psi''(w)}{\Psi(w)} = \left(\frac{\Psi'(w)}{\Psi(w)}\right)^2 + \frac{d}{dw}\left(\frac{\Psi'(w)}{\Psi(w)}\right) \tag{116}$$

Next, taking logarithms of (12) gives

$$\ln\Psi(w) = (1-\Theta-\rho)\ln w + \rho \ln\left[(1-\tau^w)w-b\right] \tag{117}$$

Differentiating (117) w.r.t w gives

$$\frac{\Psi'(w)}{\Psi(w)} = \frac{1-\rho-\Theta}{w} + \rho\frac{1-\tau^w}{(1-\tau^w)w-b} \tag{118}$$

Differentiating (118) w.r.t w gives

$$\frac{d}{dw}\left(\frac{\Psi'(w)}{\Psi(w)}\right) = -\frac{1-\rho-\Theta}{w^2} - \rho\left[\frac{1-\tau^w}{(1-\tau^w)w-b}\right]^2 \tag{119}$$

(118) may be written as

$$\frac{\Psi'(w)}{\Psi(w)}w = 1-\rho-\Theta + \rho\frac{(1-\tau^w)w}{(1-\tau^w)w-b} \tag{120}$$

and (119) may be written as

$$w^2 \frac{d}{dw}\left(\frac{\Psi'(w)}{\Psi(w)}\right) = -(1-\rho-\Theta) - \rho\left[\frac{(1-\tau^w)w}{(1-\tau^w)w-b}\right]^2 \tag{121}$$

Substitute (120) and (121) into (116) to get

$$\frac{\Psi''(w)}{\Psi(w)}w^2 = \left[1-\rho-\Theta + \rho\frac{(1-\tau^w)w}{(1-\tau^w)w-b}\right]^2 - (1-\rho-\Theta) - \rho\left[\frac{(1-\tau^w)w}{(1-\tau^w)w-b}\right]^2 \tag{122}$$

Define $A \equiv \Theta - 1 - \rho$ and $B \equiv \dfrac{(1-\tau^w)w}{(1-\tau^w)w-b}$, then (122) becomes

$$\frac{\Psi''(w)}{\Psi(w)}w^2 = (\rho B - A)^2 + A - \rho B^2 \tag{123}$$

and (120) becomes

$$\frac{\Psi'(w)}{\Psi(w)}w = \rho B - A \tag{124}$$

Thus Ψ is decreasing for $B < A/\rho$. Denote w^*: $B(w^*) = A/\rho$, and write $B = A/\rho - \varepsilon$, so that when $\varepsilon = 0$, we have $w = w^*$ then (123) becomes

$$\frac{\Psi''(w)}{\Psi(w)}w^2 = \rho^2\varepsilon^2 + A - \rho(A/\rho - \varepsilon)^2$$
$$= -\rho(1-\rho)\varepsilon^2 - \frac{A}{\rho}(A-\rho) + 2\varepsilon A \tag{125}$$

Since $A - \rho = \Theta - 1 > 0$, $\Psi'' < 0$ for $\varepsilon \leq 0$, i.e. for $B \geq A/\rho$. Thus for $w \leq w^*$, Ψ is concave, and for $w > w^*$, Ψ is decreasing (i.e. $\Psi' < 0$). Thus Ψ attains a global maximum at $\Psi' = 0$.