Sovereign Default with Unobservable Physical Capital[*]

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Abstract

This paper contributes to the literature on sovereign debt default by modelling imperfect information about borrowers' physical capital in the strategic decision-making process between borrowers and lenders. When only limited information about a borrower's capital investment is available, lenders estimate the default probability of sovereign borrowers from current output and the amount selected for debt. Borrowers will make decisions on consumption and investment, strategically, with an option to default. We calibrate our model to the Argentine economy and simulate the effect of productivity shocks between 1980 Q1 and 2017 Q4. We compute the dynamics of equilibrium bond prices, unobservable physical capital, debt and consumption in addition to equilibrium default when the lender has limited information about the borrower's capital. As capital is unobservable we show that borrowing for consumption can be optimal under certain circumstances. This is a novel result as previous literature models complete bond contracts and therefore borrowing for investment is always optimal for the borrower. We also find that our model better fits Argentina's default experience than existing models in the literature. The paper therefore highlights the importance of modelling imperfect information for understanding the behaviour of international borrowers and lenders and informing international debt policies

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1 Introduction

We provide a new theoretical framework for sovereign default when the borrower's physical capital is unobservable. This framework allows us to analyse whether it is optimal for a country to borrow for investment or consumption. The canonical position of lenders (e.g. IMF see Abbott et al. (2010)) is that foreign assets should be used for investments or for promoting long-run economic growth and in turn for generating returns to be used for debt repayment. Nevertheless the experience of several countries, such as Argentina, Nigeria and more recently Greece whose borrowing was not followed by the agreed level of investment programmes, opens up the issue for debate[1].

Previous literature has either not modelled a borrower's physical capital (Aguiar and Gopinath, 2006; Arellano, 2008) or assumed that it is fully observable by lenders (Bai and Zhang, 2011; Gordon and Guerrón-Quintana, 2013). With fully observable capital, it is possible to write a contract between borrowers and lenders and typically the resulting use of foreign assets would be for investment rather than for consumption. Furthermore, the existing literature does not provide a good fit with real world data in relation to equilibrium debt to GDP ratio, especially for Latin America and Africa (Reinhart and Rogoff, 2008).

In this paper, the modelling of the borrower side follows the sovereign default literature in allowing the debtor to decide on the future level of capital, consumption and debt with an option to default. However, differently from the

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[1]Easterly (2005) shows the evidence of structural adjustment loans (SAL) provided by IMF and World Bank to developing countries. The program aims to sustain the long-run economic growth, however non of the top repeated recipients could achieve macroeconomic conditions.

literature, borrower's capital investment cannot be observed by the lender. The lender cannot observe physical capital, and therefore the pricing schedule does not depend on capital. The borrower can therefore choose to borrow for consumption rather than for investment in capital, without altering the price of borrowing.

We calibrate our model to the Argentine economy and simulate the effect of productivity shocks between 1980 Q1 and 2017 Q4. We show results in terms of bond prices and a borrower's capital accumulation, debt and consumption in addition to default options at the steady state and on the transition path with productivity shocks[2].

We also find that at below the steady-state level of physical capital, the borrower will find optimal to use the foreign assets for investment, while if capital is above the steady state, the borrower will sell the physical capital for consumption. Furthermore, the level of capital will steadily move towards the steady-state. Notably, if the borrower holds a large amount of debt and low capital, she(he) will not be able to borrow further and will stay at below the steady-state level of capital. Furthermore, with a negative productivity shock, the borrower will be able to smoothen consumption through sacrificing capital, if the initial capital is at or above the steady-state level. On the other hand, if the initial capital is below the steady-state level, the borrower is unable to smoothen consumption through borrowing more or selling physical capital. The simulation results therefore show that it is not optimal to force debtors to use foreign assets to invest only, as under certain conditions, consumption is the optimal decision. These results have not

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[2]As in the previous literature (Arellano, 2008; Bai and Zhang, 2011; Gordon and Guerrón-Quintana, 2013), we only model productivity shocks in our paper for simplicity and comparability with other papers.

been captured by the previous literature.

In addition, our model is better suited to fit the default experience of several countries especially in Latin America and Africa than existing models in the literature by showing higher debt to GDP ratio at the steady-state.

Our results therefore confirm the importance of imperfect information for understanding the behaviours of international borrowers and lenders and informing international debt policies.

This paper is structured as follows: Section 2 contains a review of previous theoretical literature on sovereign default; our theoretical model of sovereign default with unobservable physical capital is described in Section 3; the data and calibration are presented in Section 4, while the quantitative results and the Monte Carlo simulations are analysed in Section 5; section 6 concludes the paper.

2   Related Literature

Over the last decade, global financial sectors have become more integrated and have shared risk throughout international credit markets. A government's decision to declare default has become more important to the world's economy and several researchers have demonstrated the reasons for sovereign default by using theoretical models. In 1981, the theoretical framework of Jonathan Eaton and Mark Gersovitz became a well-known study of borrowing with default when they published their paper assuming that the decision to default is an optional strategy for debtor countries.

A seminal framework of an endogenous relationship between default probability and

output shocks was initially provided by Eaton and Gersovitz (1981) and developed by Arellano (2008). In these models borrowers strive to maximise consumption by using foreign assets with imperfect information. Therefore, they find that the borrowers will borrow more during bad times and repay during good times (counter-cyclical policy). In this framework, international debt is an endogenous variable in the resource constraint depending on the shocks. In addition, the authors also add the probability of re-entry into the default function as an exogenous variable. Sovereign debtors can re-enter international credit markets after some periods of debt exclusion; this assumption is consistent with recent empirical evidence that most defaulting durations are between three and six years (Reinhart and Rogoff, 2008; Sandleris et al., 2004).

Bai and Zhang (2010), Bai and Zhang (2011), Gordon and Guerrón-Quintana (2013), Liu and Shen (2016) and Park (2017) include observable physical capital in their models. In these papers, physical capital is assumed as being optional assets that are fully observed, as well as bonds. The government could then choose to invest through two types of asset, bonds and physical capital with the total amount of bonds allowed to be negative (net borrowing). If the government invests more in physical capital, it can enhance output; however, physical capital will depreciate every year. The role of lenders is implicit in these models as the market can fully observe borrowers and always accept the requested amount of borrowing by adjusting the risk premium, providing borrowers do not default in the next period. Thereby, perfect information about physical capital in this standard of literature provides a direct incentive for borrowers to increase capital investment in order to reduce the risk premium.

Liu and Shen (2016), present a positive relationship between capital investment and bond prices, where higher accumulated capital decreases risk-premium and increases bond prices. Park (2017) raise an issue concerning the use of two correlated assets; foreign asset and domestic capital in the model. If borrowers hold high physical capital, they are allowed to borrow more from an international credit market and will enjoy a lower risk-premium. Consequently, borrowers invest more and borrow more. Therefore, countries will have accumulated physical capital above the optimal level.

Our paper expand upon this literature by assuming that the borrower's physical capital is not observed by the lender. This allows us to capture the effects of imperfect information on the the outcome of the negotiation between borrowers and lenders.

3   The Model Economy

In this section, we provide a new model of borrower's default decision when capital is unobservable. It will be used to derive the optimal values of physical capital and debt in response to positive and negative productivity shocks.

3.1   Consumption Preference

Both borrower and lender have the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{3.1}$$

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{3.2}$$

where c is consumption, σ is the risk aversion rate and β is the discount factor. As standard in the literature, utility function assumes the constant relative risk aversion (CRRA) specification.

3.2 Production Functions

Production is assumed to take the following Cobb-Douglas specification:

$$y = ak^{\alpha}, \tag{3.3}$$

where y is output, k is physical capital and a is total factor productivity (TFP) shock. An increase in the level of capital accumulation (k) positively influence output (y). In addition, the TFP shock for normal periods is assumed to follow AR(1) stochastic process with the gaussian distribution[3]:

$$a_t = \mu + \rho\, a_{t-1} + \epsilon_t\,, \tag{3.4}$$

where $\mu = 1$, $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\rho \leq 1$. If the country defaults, the borrower will be excluded from the international credit market and penalised. The Cobb-Douglas production function becomes:

$$y^{def} = a^{def}k^{\alpha}, \tag{3.5}$$

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[3]We discretise a continuous stochastic process of TFP by using quadrature method (Tauchen, 1986; Tauchen and Hussey, 1991) (see Appendix 7.2)

where a^{def} is TFP shock during the default period and y^{def} is output during default with $y^{def} < y$. Stylised fact from defaulting countries confirm that output tends to drop below the trend[4] during default periods (Reinhart and Rogoff, 2011; Tomz and Wright, 2007).

Hence, there is a default cost in terms of outpuxt as a penalty, if the government decides to default. The boundary for the TFP shock during repayment and default periods can be written as: $a \in (\underline{a}, \overline{a})$ and $a^{def} \in (\underline{a}, \overline{a}^{def})$ respectively; where $\overline{a}^{def} \leq \overline{a}$. Output during the penalty phase will drop by the default output cost (χ) below the trend. If $\chi = 0$, there will be no default cost and $a = a^{def}$.

3.3   The Resource Constraints of the Sovereign Borrower

In a closed economy, a government has only one type of assets and can only decide on the next period's physical capital (k'). The country's resource constraint in terms of consumption is based on the level of output which also depends on the choice of capital investment. If the government decides to invest more in capital, there will be less resources left to consume in the current period and higher capital depreciation in the following period. On the contrary, selling physical capital instantly increase consumption and will decrease capital depreciation in the following period.

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[4]The trend is defined by an average growth rate of the past GDP (Tomz and Wright, 2007)

3.3.1   Closed Economy

Based on the assumptions, the resource constraint for a closed economy is given as follows:

$$c = y + (1 - \delta)k - k' - \Phi(k', k), \tag{3.6}$$

where c is consumption, y is output, k is current physical capital and k' is the next period's physical capital. Besides, capital depreciation and capital adjustment cost are denoted by δ and $\Phi(k', k)$ respectively. Noticeably, a change in domestic capital accumulation has an endogenous relationship with output through the Cobb-Douglas production function.

3.3.2   Open Economy

A country with access to the international financial market has the additional option to borrow or invest in international financial assets. In this way, the government has two available type of assets; foreign assets and physical capital that can be used to maximises utility. Hence, the resource constraint for a small open economy can be expressed as following:

$$c = y + b - q(b', a)b' + (1 - \delta)k - k' - \Phi(k', k), \tag{3.7}$$

where $q(b', a)$ is bond price and, b and b' are current and next period's bond. In an open economy, the government decide upon the levels of both k' and b' to maximise utility. However, bonds are only available during repayment periods as borrowers have to continue repayment in order to get future access to the international financial

market. Nevertheless international lenders have no power to either force a sovereign defaulter to repay or to seize assets.

3.4 Decision Function of the Sovereign Borrower

Next, the sovereign borrower must decide whether to repayment or default. If the sovereign borrower chooses to default, they will be forced to enter the financial autarky with a tendency for lower output. A sovereign defaulter will be unable to enter the international financial market for a certain period after a default decision. For all periods, the government has to decide between repayment and default by taking into account all possible outcomes after the decision. Hence, the government decision function is derived as follows:

$$v_g^0(b, k, a) = \max_{\{d,r\}}\{v_g^d(k, a), v_g^r(b, k, a)\}, \qquad (3.8)$$

where v_g^0 is the present value of the government decision, v_g^d is the value of the default decision and v_g^r is the value of the repayment decision. The government will choose whether to default or repay based on the valuation of these two decisions' functions. For instance, the government will choose to repay their debt if the value of the repayment decision is greater than the value of default. From choosing repayment, a sovereign borrower can continue to enter the financial market; however, if the government avoids repayment, they will be punished by exclusion from the international financial market and tend to return after a period.

3.4.1 Value Function of Default

Therefore, the default function can be derived as the following:

$$v_g^d(k, a) = \max_{\{k'\}} u(c) + \beta \, \mathbb{E}[\theta v_g^0(0, k', a') + (1 - \theta)v_g^d(k', a')], \qquad (3.9a)$$

subject to:

$$c = y^{def} + (1 - \delta)k - k' - \Phi(k', k), \qquad (3.9b)$$

and

$$c, k' \geq 0, \qquad (3.9c)$$

where y^{def} is output during default periods $(y^{def} \leq y)$ and θ is the probability of re-entry into the international financial market.

3.4.2 Value Function of Repayment

Next, the repayment function is given by:

$$v_g^r(b, k, a) = \max_{\{b', k'\}} u(c) + \beta \, \mathbb{E} \, v_g^0(b', k', a'), \qquad (3.10a)$$

subject to:

$$c = y + b - q(b', a)b' + (1 - \delta)k - k' - \Phi(k', k), \qquad (3.10b)$$

and

$$c, k', q(b', a) \geq 0. \qquad (3.10c)$$

From the above government decision function, it is obvious a sovereign borrower will decide to default when the value of the default function is greater than repayment. They will therefore enter into the penalty phase with the probability to re-enter into the international market at θ.

3.5 Capital Adjustment Cost

In addition, the capital adjustment cost $\Phi(k', k)$ has a standard quadratic form that can be shown as follows:

$$\Phi(k', k) = \frac{\Phi}{2} \left(\frac{k' - (1 - \delta)k}{k} \right)^2 k, \tag{3.11}$$

where Φ is set at 3 or 5 in general real business cycles (RBC) literature to match the output moments (Bai and Zhang, 2012; Gordon and Guerrón-Quintana, 2013). This paper assumes that physical capital can be traded anytime with a capital adjustment cost even in the default periods. On the other hand, a financial asset or bond is only available during the repayment periods, based on the borrower's default probability.

3.6 Bond Price Schedule

The bond price schedule can be characterised by the borrower's default probability as shown below:

$$q(b', a) = \frac{1 - \Psi(b', a)}{1 + r}, \tag{3.12}$$

where Ψ is the default probability of a sovereign borrower and r is the risk-free rate. In this paper, the bond price schedule is provided by an international lender. Under

an incomplete market, a sovereign borrower can borrow only the amount and price given from an international lender. For example, a sovereign borrower will be unable to borrow if an international lender does not agree the bond price and amount.

Moreover, under the international sovereign bond market, the lender cannot commit the next period's physical capital of the sovereign borrower. Especially, the physical capital cannot be perfectly observed outside the country. We assume the asymmetric information between a sovereign borrower and an international lender; consequently, physical capital cannot guarantee the repayment ability of a sovereign borrower. The international lender will obtain the default incentive of a sovereign borrower by observing only the bond (b,b') and the TFP shock (a) of the sovereign borrower. Thereby, the default probability of a sovereign borrower (Ψ) and the bond price schedule $q(b', a)$ are given from the international lender's viewpoint that will be discussed in the following section.

3.7   Resource Constraints of the Foreign Lender

Under the international lender's viewpoint, the endowments of a closed economy and a small open economy are different from the sovereign borrower's viewpoints because physical capital is unavailable to be observed and committed in the model. Hence, the international lender views the resource constraints of sovereign borrowers under the closed economy as below:

$$c = y^{def}, \tag{3.13}$$

and the small open economy is given by:

$$c = y + b - q(b', a)b', \tag{3.14}$$

where k and k' are unobservable and omitted in the model. Gordon and Guerrón-Quintana (2013) demonstrated that physical capital of sovereign defaulters within the border cannot be taken by foreign lenders in the modern economy. Physical capital cannot be collateral to secure bonds issued by sovereign borrowers.

Moreover, providing bond price schedule to borrowers from their rational behaviour of future capital investment is practically troublesome. Physical capital is difficult to measure and, foreign lenders cannot force sovereign borrowers to commit on the capital investment. Therefore, the future capital investment is excluded from the lender's view in the negotiation of bond price with sovereign borrowers.

3.8 Decision Function of the Foreign Lender

Foreign creditors will decide to lend money to sovereign borrowers based on their default's incentive in the future period. The decision functions under the lender's viewpoint are given by:

$$v_l^0(b, a) = \max_{\{d,r\}}\{v_l^d(a), v_l^r(b, a)\}, \tag{3.15}$$

where $v_l^0(b, a)$ is the value of the decision function from the international lender.

3.8.1 Value Function of Default

Next, the value function of the default decision can be expressed as follows:

$$v_l^d(a) = u(c) + \beta\, \mathbb{E}[\theta v_l^0(0, a') + (1 - \theta)v_l^d(a')] \tag{3.16a}$$

subject to:

$$c = y^{def}, \tag{3.16b}$$

and

$$c \geq 0, \tag{3.16c}$$

where y^{def} is obtained from observing the current output during penalty periods and which is equal to y^{def} under the borrower's view.

3.8.2 Value Function of Repayment

Besides, the repayment function takes the form of the international lender's view as follows:

$$v_l^r(b, a) = \max_{\{b'\}} u(c) + \beta\, \mathbb{E}\, v_l^0(b', a'), \tag{3.17a}$$

subject to:

$$c = y + b - q(b', a)b', \tag{3.17b}$$

and

$$c, q(b', a) \geq 0. \tag{3.17c}$$

where y is also obtained from observing the borrower's current output which is the same as y under the sovereign borrower's aspect. This paper assumes that the

international lender can observe the TFP shock as well as the sovereign borrower such that the output can be expressed in the production function with a constant value of current physical capital and the TFP shock where $y = ak^{\alpha}$.

3.9 Default Probability in Equilibrium

The sovereign borrower and the international lender share the same bond price schedule under the above value functions from the lender's view. Thus, sets of repayment and default decisions under the lender's viewpoint are given by:

$$R(b) = \{a \in A : v_l^r(b, a) \geq v_l^d(a^{def})\}, \tag{3.18}$$

$$D(b) = \{a \in A : v_l^r(b, a) < v_l^d(a^{def})\}, \tag{3.19}$$

where $R(b)$ is the repayment set and $D(b)$ is the default set under the lender's aspect(v_l^0). Next, the default set under the lender's view can be achieved as:

$$\Psi(b', a) = \int_{D(b')} f(a', a) \, da' \tag{3.20}$$

where $f(a', a)$ is the stochastic process provided in Equation (3.4). It is obvious that $0 \leq \Psi \leq 1$. When $D(b') = \emptyset$, the default probability is zero. From the above recursive equations, the sovereign borrower will choose the amount of assets b' and k' after recognising the current TFP shock (a). Simultaneously, the international lender will obtain b' and a from the sovereign borrower and return the bond price (q) based on the decision function (v_l^0). Finally, the set of all possible outcomes (v_g^0) with policy functions of physical capital, debt and default decisions with the corresponding TFP shock can be obtained.

4   Data and Calibration

The model is calibrated to target the Argentine economy by using quarterly data from 1980 Q1 to 2017 Q4 with some parameters set based on recent RBC literature. The main contribution is to explain the interactions among physical capital, debt, output, and risk premium and default probability. We use a discrete model with Monte Carlo simulation and the moment matching method in order to explore and fit the model to actual data. As a result, the model in this paper can provide a closer fit to the average actual level of debt and consumption ratio as well as the moments and the correlation coefficients.
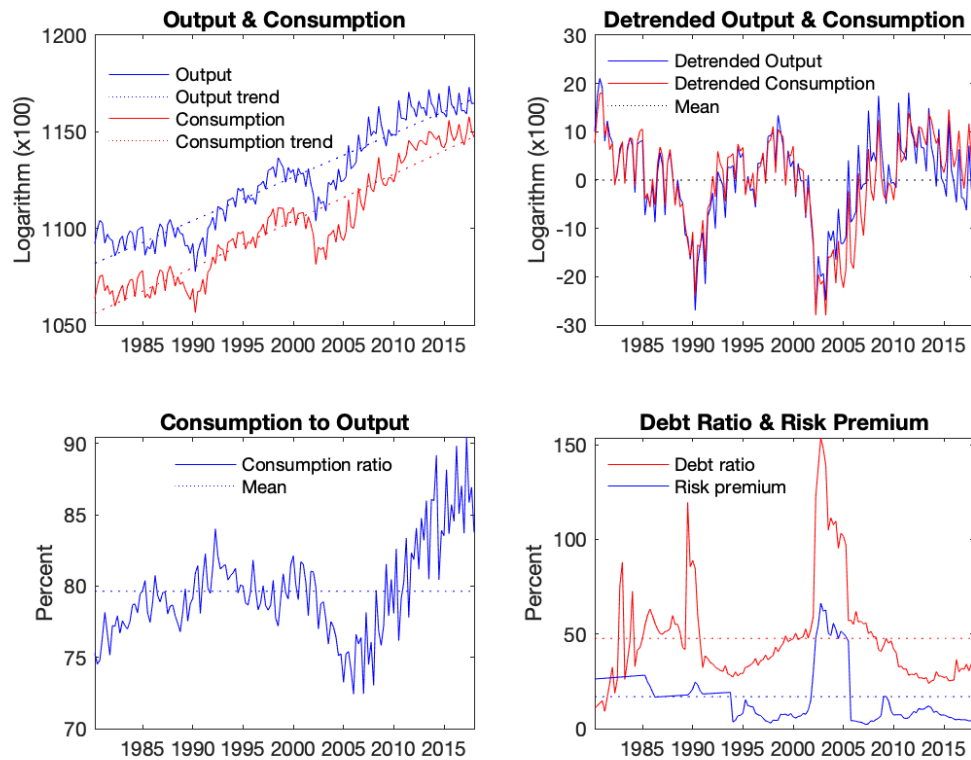

Figure 1: Output, Consumption, Trend and Debt

Data are from the Argentine Ministry of Economy (MECON). The data are available quarterly, non-seasonally adjusted and at constant 2010 U.S. dollars, starting from

the first quarter of 1980 to the last quarter of 2017 over 152 periods. Figure 1 shows plots of output, consumption, debt, risk premium and trends. GDP and consumption are in log and linearly de-trended. Figure 1 clearly shows an upward trend for both output and consumption in Argentina with a standard deviation for consumption and output of 0.0969 (σ_c) and 0.0917 (σ_y) respectively.

Thus, consumption is 1.07 times more volatile than output (σ_c/σ_y). Moreover, there is a significant and positive correlation between output and consumption equal to 0.9223 (ρ_{cy}). In Figure 1, the consumption to output ratio of Argentina was 75% in 1980 Q1, and peaked at 90% in 2017 Q4. Besides, mean and standard deviation of the consumption ratio over 38 years are 79.63% $(\mu_{c/y})$ and 0.0336 $(\sigma_{c/y})$ respectively.

Table 1 below presents business cycle statistics for the Argentina's economy between 1980 Q1 and 2017 Q4. The first-order auto-regression model of output without a constant term has 0.6786 (ρ_y) auto-correlation. Next, we use the ratio of external debt to GDP to measure the external debt level in the model. Table 1 presents the debt ratio with a mean of 47.67% (μ_b) and standard deviation of 0.2719 (σ_b). before the late 1980s, the external debt to output ratio of Argentina had never exceeded 50%. It reached the bottom value at 9.07% in 1981 Q1 and increased to a peak of 153.63% in 2002 Q3. Spikes were clustering around the period of financial crisis and recession in 1990 and 2003. Noticeably, there is a negative correlation coefficient between output and debt ratio of -0.6552 (ρ_{by}). This negative relationship was obviously seen in 1990 and 2002 when the debt ratio increased to its peak at the same time as output experienced the biggest drop in Argentina.

Table 1: Argentina's business cycle statistics (1980 Q1 - 2017 Q4)

Parameters	Values	Parameters	Values
μ_y	0	σ_y	0.0917
μ_c	0	σ_c	0.0969
$\mu_{\frac{c}{y}}$	0.7963	$\sigma_{\frac{c}{y}}$	0.0336
μ_b	0.4767	σ_b	0.2719
μ_ψ	0.1689	σ_ψ	0.1420
$\frac{\sigma_c}{\sigma_y}$	1.0708	$\frac{\sigma_{c/y}}{\sigma_y}$	0.3664
$\frac{\sigma_b}{\sigma_y}$	2.9651	$\frac{\sigma_\psi}{\sigma_y}$	1.5485
ρ_y	0.6786	ρ_c	0.7860
ρ_b	0.8797	ρ_ψ	0.9440
ρ_{cy}	0.9223	ρ_{by}	-0.6552
$\rho_{\psi y}$	-0.4713	$\rho_{b\psi}$	0.6953
ρ_{cb}	-0.7616	$\rho_{c\psi}$	-0.542

Table 1 also shows that debt is 2.97 (σ_b/σ_y) times more volatile than output and 2.81 (σ_b/σ_c) times than consumption. The risk premium is obtained from Argentina's overall debt.

In Table 1, the mean and standard deviations of risk premium are 0.1689 (μ_ψ) and 0.1420 (σ_ψ) respectively. The correlation coefficient between output and risk premium is negative at -0.4713 $(\rho_{\psi y})$ whilst, debt ratio and risk premium have a positive correlation at 0.6953 $(\rho_{b\psi})$. Overall, the signs of correlation coefficients for Argentina are consistent with the RBC literature and stylised facts from developing countries (Aguiar and Gopinath, 2006; Yue, 2010).

Table 2 induces over model parameters for the Argentine economy. The discount factor (β) is borrowed from the sovereign default literature (Arellano, 2008) and it is equal to 0.35. Risk aversion (γ_g, γ_l), risk-free rate (r), capital share (α) and capital depreciation (δ) are 2, 1% and 7% respectively, based on the RBC literature for developing countries.

Table 2: Model specific parameter values

Parameters		Values
Discount factor	β	0.95
Risk aversion of borrower	γ_g	2
Risk aversion of lender	γ_l	2
Risk-free rate	r	0.01
Capital share	α	0.35
Capital depreciation	δ	0.05
Output default cost	χ	7%

We calibrate other parameters of interest as shown in Table 3. The stochastic structure of TFP shock, which follows the AR(1) process, is specified by using the moment matching method to target the Argentine economy (see Table 4). ρ_a and σ_ϵ are found to be 0.96 and 0.015 respectively. This stochastic structure fits the actual volatility of output (σ_y) and consumption (σ_c) and correlation coefficient between them (ρ_{cy}) at 0.0917, 0.0969 and 1.0708 respectively. The probability of re-entry (θ) and output default cost (χ) are fixed at 5% and 7% consistently in the literature; Following the standard quadratic form of capital movement (Bai and Zhang, 2011; Gordon and Guerrón-Quintana, 2013), capital adjustment cost is set at 1.4.

Table 3: Calibration parameters

Parameters		Values
Stochastic structure (TFP)	ρ_a, σ_ϵ	0.978 , 0.011
Probability of reentry	θ	5%
Capital adjustment cost	Φ	1.4

Moreover, relative consumption and output volatility ($\frac{\sigma_c}{\sigma_y}$), average consumption ($\mu_{\frac{c}{y}}$) and debt ratio ($\mu_b$) are calibrated over the specified periods to be 1.0708 times, 79.63% and 47.67% respectively (see Table 4 below).

Table 4: Target statistics

Parameters		Values
Output volatility	σ_y	0.0917
Consumption volatility	σ_c	0.0969
Consumption output volatility	$\frac{\sigma_c}{\sigma_y}$	1.0708
Correlation coefficient (c,y)	ρ_{cy}	0.9223
Average consumption ratio	$\mu_{\frac{c}{y}}$	79.63 %
Average debt ratio	μ_b	47.67 %
Default periods		12.5 %

Furthermore, this paper uses a discrete model and follows Tauchen's method[5] for obtaining a finite state Markov chain approximation of the nonlinear asset (Tauchen, 1986). Hence, the TFP shock with the above stochastic structure is discretised into a 47-state Markov chain.

The possible length of debt and physical capital in the grids are specified from the Argentine economy between 1980 Q1 and 2017 Q4 as $\{\underline{b}, \overline{b}\} = \{-4.1, 0.5\}$ and $\{\underline{k}, \overline{k}\} = \{1, 12\}$ respectively. From these lengths of the grids, debt ratio lies between $\frac{\underline{b}}{y}$ and $\frac{\overline{b}}{y}$, whilst capital ratio lies within $\frac{\underline{k}}{y}$ and $\frac{\overline{k}}{y}$. Hence, the upper and the lower bounds are set to cover the possible length of debt and capital with respect to the past experience of the Argentina's economy. Specifically, the total number of grids for debt and physical capital are 47 and 111. The 47-finite state of debt is equally distributed between the interval $\{\underline{b}, \overline{b}\}$, while the grid of capital is set to cluster around the expected steady state [6].

Finally, we simulate the model using the Monte Carlo simulation over 100,000 times.

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[5]This method has been widely used by the previous literature on nonlinear model with a discrete-valued Markov chain see (Aguiar and Gopinath, 2006; Arellano, 2008; Gordon and Guerrón-Quintana, 2013; Liu and Shen, 2016; Schaltegger and Weder, 2015)

[6]The total number of grids for capital is 111. There are 10 state spaces that equivalently located with 0.5 distance between the value of capital at the intervals $\{1, 3.5\}$ and $\{10.5, 12\}$. Beside, other 101 state spaces are equivalently located within the interval $\{4, 10\}$. This setup helps to reduce the number of time spending in the computation and the distance between grid around the steady state of capital; the optimal choice of capital tends to cluster around the steady state.

5   Quantitative Results

This section will provide the result of the stimulation and their discussion. It includes three main parts. The first part will define the equilibrium bond price schedule, default probability and policy functions of a sovereign borrower. The second part will analyse impulse response functions and the Monte Carlo simulation. Finally, the third part will provide a comparison between actual data for the Argentina's economy from 1980 Q1 to 2017 Q4 and the model simulations.
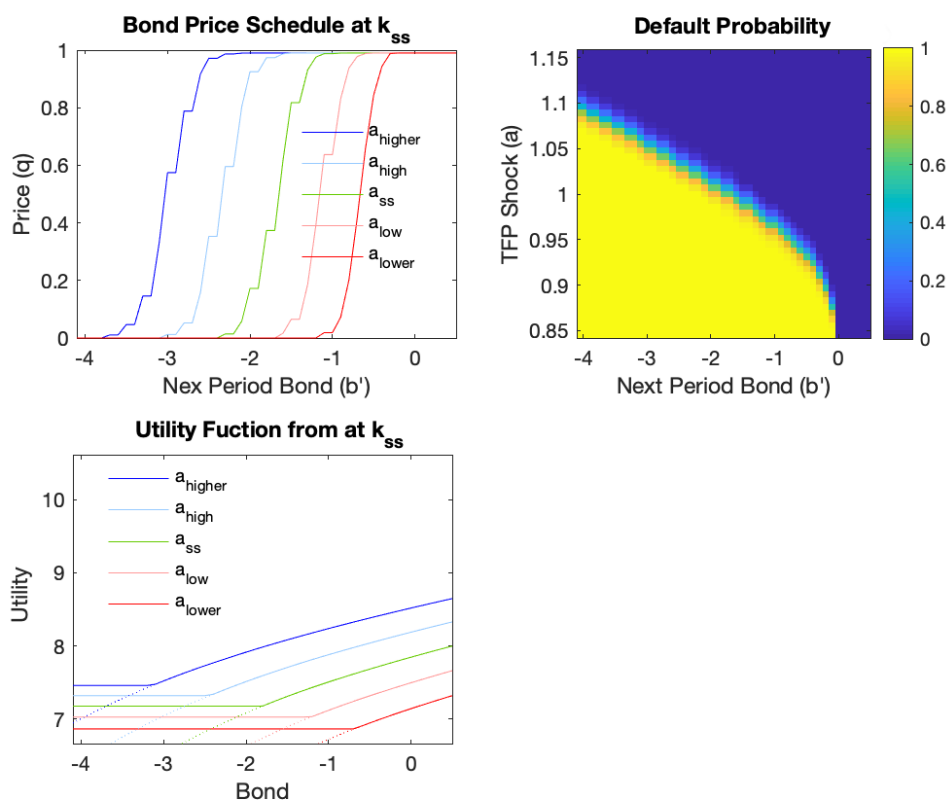

Figure 2: Bond Price Schedule, Default Probability and Utility Functions

The bond price schedule at the steady-state level of capital is given in Figure 2. We can derive an equilibrium a bond price (q) schedule that responds to the next period's bond (b') and the current level of output (y) in turn affected by current capital (k) and technology shock (a). The plot of the bond price schedule indicates

the negative relationship between next period's level of debt and bond price. An international lender will respond to a borrower's selection of debt amount and the current period's level of output. If a country intends to borrow large amount of debt, the risk premium increase in the international market and the borrower will be charged a high price. The opposite is true if the borrower selects a lower amount of debt. Moreover, Figure 2 also shows differences in bond price schedules for several technology shocks where a_{higher}, a_{high}, a_{ss}, a_{low} and a_{lower} represent TFP percentage changes of 5.5%, 2.75%, 0%, -2.75% and -5.5% from the steady-state value (calculated without shocks).

The sovereign country can borrow a higher amounts of debt at a lower risk premium with a positive TFP shock. On the other hand, a negative TFP shock can lead to an instant increase in bond risk premium for the same amount of debt. Thereby, a large negative shock can instantly induce a sovereign country with large debt to default because of an expensive borrowing rate (higher risk premium) to be faced in the following period.

The future default probability is shown as a heat map chart in Figure 2. The figure is plotted from three dimensional data for bond selection (b'), TFP shock (a) and default probability (Ψ) where the default probability lies between zero (blue) and one (yellow). From the plot, if a country intends to save with a bond (b') above or equal to zero, there will be no default in the following period. With $\Psi = 0$, the country can borrow or save money at the risk-free rate (without risk premium; see the blue shade area in Figure 2). However, if the country intends to borrow, there will be a positive default probability (the yellow area) in the next period with respect to the size of the TFP shock. With a negative shock, the borrower will

be charged at a higher risk premium in the following period because of a positive default probability. For a default probability of 100%, the borrower will be unable to borrow on the international market.

Furthermore, Figure 2 also shows the utility functions at the steady-state level of capital with various TFP shocks and debt. The solid lines in the figure are optimal utility values (depending on repayment and defaults). From the figure, the utility tends to be higher if there is a positive TFP shock in the current period. For the same value of TFP, the utility will be higher with more current bonds or foreign assets (b). In addition, the dotted lines in Figure 2 indicate the utility values when the sovereign borrower continues to repay debt instead of choosing to default. The utility values with a default decision will be flat as bond selection is unavailable for default countries. From a comparison between the solid and dotted lines in Figure 2 for large debt, we notice that default gives rise to higher utility. The sovereign borrower with large debt tends to default for a drop in current output or a lower value of TFP.

Figure 3 plots debt and capital functions at the steady-state level of capital (k_{ss}) and debt (b_{ss}) with positive and negative TFP shocks. In the figure, the red and blue lines indicate the policy function (choosing next period bond and capital) responding to a negative and positive TFP shocks respectively. The black line is defined when there is no shock to the economy, while the dotted line is the 45 degree line that helps to define an unchanged level of current and future period resources.
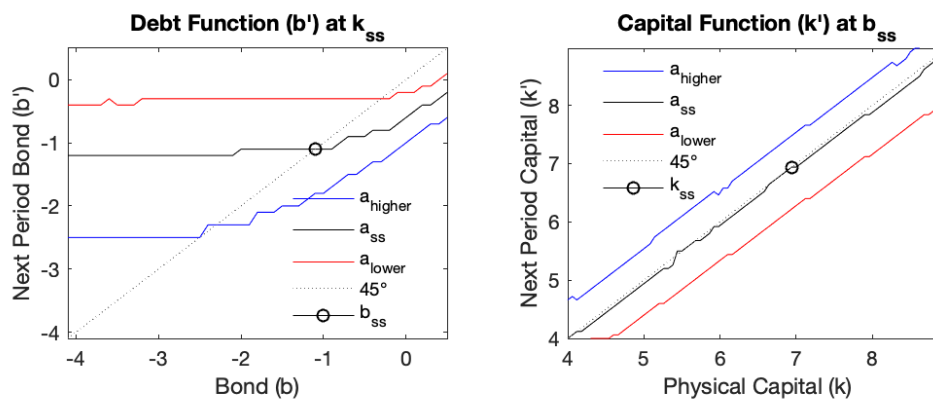
Figure 3: Policy Functions for Debt and Capital Investment

In the figure of debt function, there will be a shift from the black line to the blue line when the positive TFP shock occurs. It indicates that the sovereign borrower with current debt ($b < 0$) will choose to borrow more if a positive TFP shock occurs. The policy function will shift towards the red line when there is a negative TFP shock. It implies that the borrower will reduce debt to a certain level in response to a negative TFP shock. Moreover, the intersection of the policy line without any shock (black) and the 45° line (dotted line) indicates the steady state level of debt. If the economy starts with bond at a level above or below the steady state level of debt (i.e. the intersection), the bond function will converge to the steady stage.

The policy function for future capital with respect to its current level is also given in Figure 3. At the steady state level of debt, a sovereign borrower will increase/decrease capital investment from the positive/negative TFP shock. From a comparison of the policy line without shock (black solid line) and the 45° line (dotted line), we find the steady-state level of capital at 6.94. If the borrower start with capital above the steady-state level, the policy line will be slightly below the 45° line. This means that a sovereign borrower will decide to sell physical capital, if its current level is above the steady-state. The policy line is slightly above the 45° line when the current capital is below the steady-state level. This suggests that

the country will choose to invest more until it reaches the steady state.
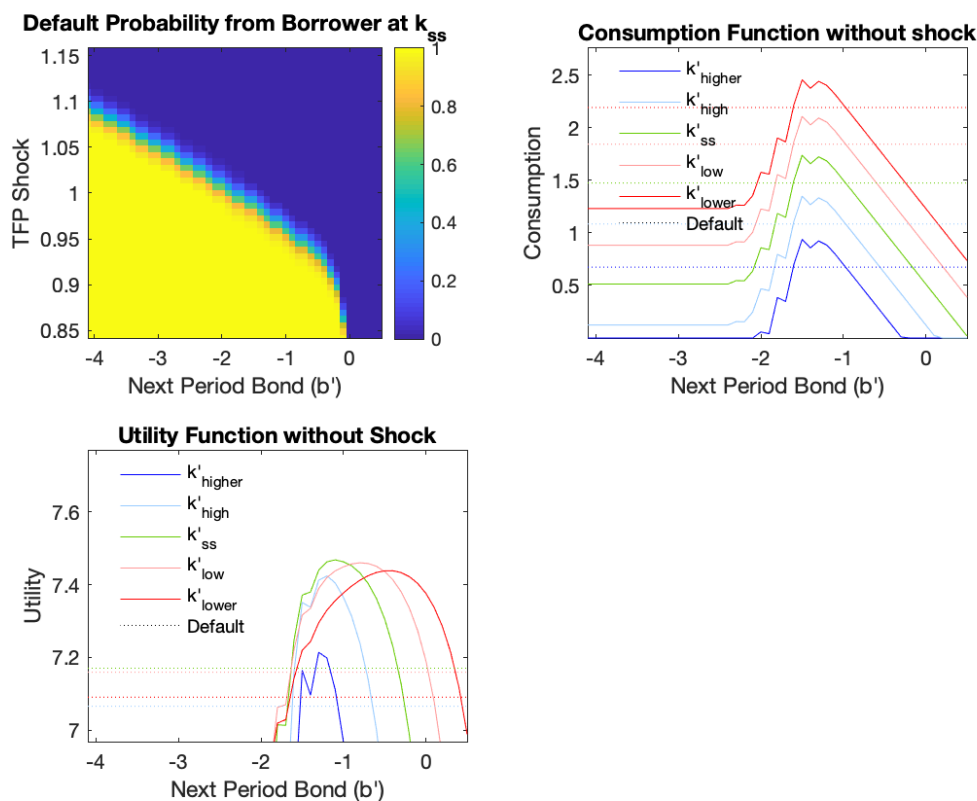

Figure 4: Default Probability, Consumption and Utility Functions of the Borrower

Figure 4 gives the heat map plot of the borrower's default probability the steady-state level of capital (k_{ss}). The future default intention of the sovereign borrower can be deduced from this plot as a function of the TFP shock (a) and the next period's bond (b'). As shown in the figure, a country with large debt has more intention to default after a drop in output (yellow area with TFP less than 1). For a TFP > 1 (blue area), debtors will remain at a high level of debt.

Figure 4 also shows plots of the consumption and utility functions with respect to the next period's bond and capital investment without shocks. The solid and dotted lines show consumption from repayment and default decisions respectively. From the consumption plot, selling physical capital can instantly relax the budget constraint, which in turn leads to an increase in current consumption. Besides

consumption, when repayment are included, peaks for next period's bond (b') equal to -1.5. Comparing between the repayment (solid lines) and default (dotted lines) decisions, the debtor is better off staying around its peak with repayment decisions. At its peak, the country will be able to consume more from borrowing, if they continue to repay and borrow from the international bond market. However, the price of bonds is also based on the default risk which will make the bond price expensive with large amounts of debt. If instead the country saves money in the bond market, it will reduce the current budget constraint and consumption.

The utility function for the next period's bond and capital investment is also given in Figure 4. The plot shows an inverted u-shape relationship between utility values and the next period's bond under repayment. The utility curve of repayment peaks at the steady-state level of debt (b'_{ss}). When we compare repayment and default decisions plot we notice the utility values of repayment at the steady state is always higher than under default because the sovereign debtor has no intention to default at the steady state, if no shocks occur. More importantly, comparing utility curves for different capital investments, we observe that capital level at the steady state (k'_{ss}) generates the highest utility value.

Figure 5 shows the dimensional heat map charts of the default probability based on the borrower's selection of capital investment (k'). Areas of default intention (yellow) in each layer clearly define the probability of default in the next period. The top layer is generated from future default intention with high capital investment, whilst the middle and bottom layers are the default probability at the steady state and low capital investment respectively. From the sliced layers in the chart, we notice that the default probability in the following period changes due to the amount of the

next period's capital (k'). An increase in physical capital will endogenously enhance productivity as well as raise the number of secured assets that can be used as a last resort during recession. Therefore, an increase in capital investment significantly reduces the default probability in the following period, as shown in Figure 5.
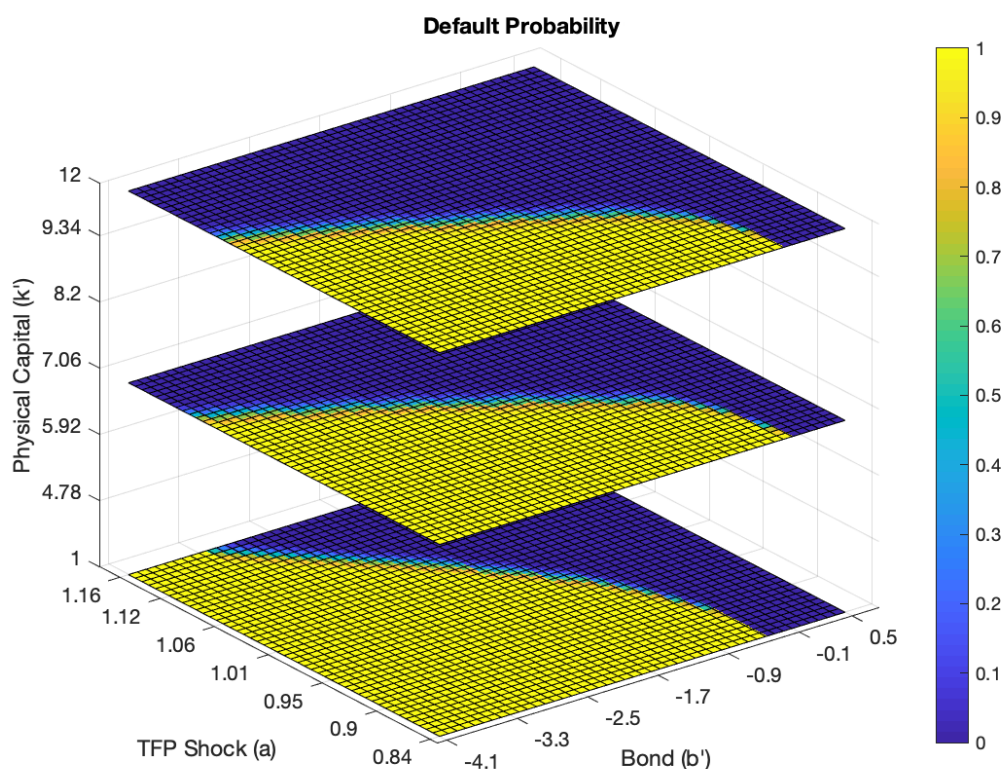

Figure 5: Default Probability from Borrower based on Physical Capital

On the other hand, a decrease in capital investment may raise current budget constraint and consumption instantly, but it tends to reduce future default probability. The debtors with lower physical capital obviously have fewer assets to sell during recession and, thereby, the future default probability is higher. More importantly, a sovereign borrower with little capital has higher probability to default with little debt compared to those with high physical capital. This characteristic of default matches the actual data according to which poor countries with low capital have a higher number of defaults (Reinhart and Rogoff, 2008).

The second part of this section will analyse the impulse response functions and the

Monte Carlo simulation. At the steady state level of capital, the impulse-response functions following a positive and a negative TFP shock of 2.75% with 0.50 persistence are given in Figure 6. Notably, figure 6 shows the response of foreign asset (b), net borrowing $(-(b' - b))$, output (y), physical capital (k), capital investment $(k' - k)$, net asset $(k + b)$, consumption (c) and net cashflow $(-\dot{b} - \dot{k})$.

At the steady state, when a positive shock occurs in period 1, the country starts to borrow more for both consumption and investment; net borrowing becomes negative, while capital investment and consumption increase. After that, the country slightly decreases capital, repays its debt and reduces consumption until the TFP shock fades. Capital investment is significantly higher than consumption during the period of borrowing. Hence, we can deduce that a country tends to borrow for investment rather than consumption during a period of positive TFP shock.
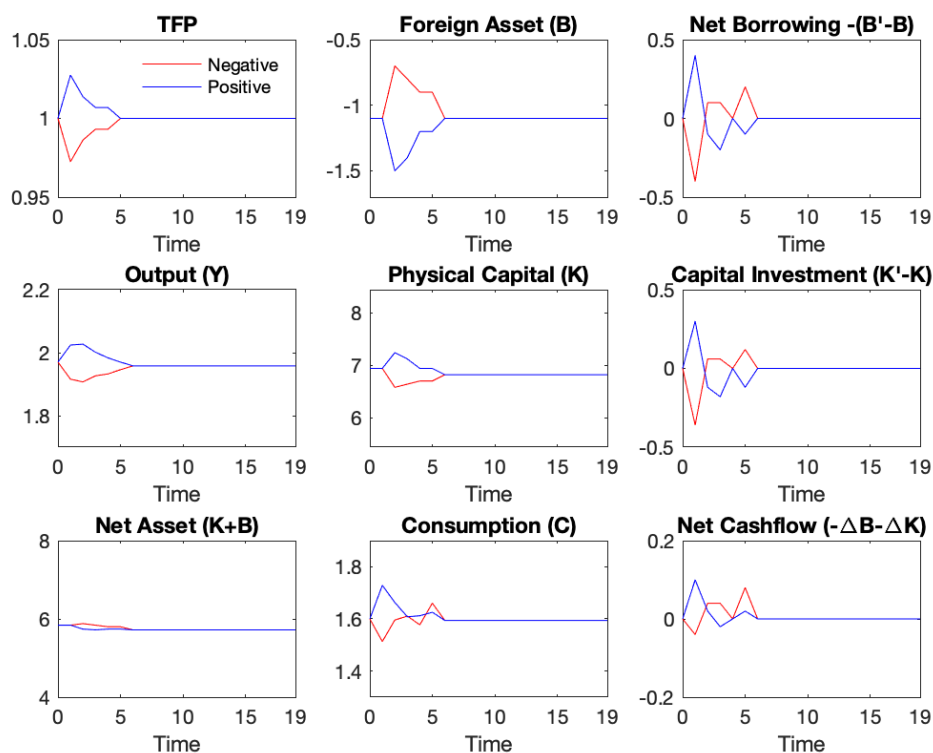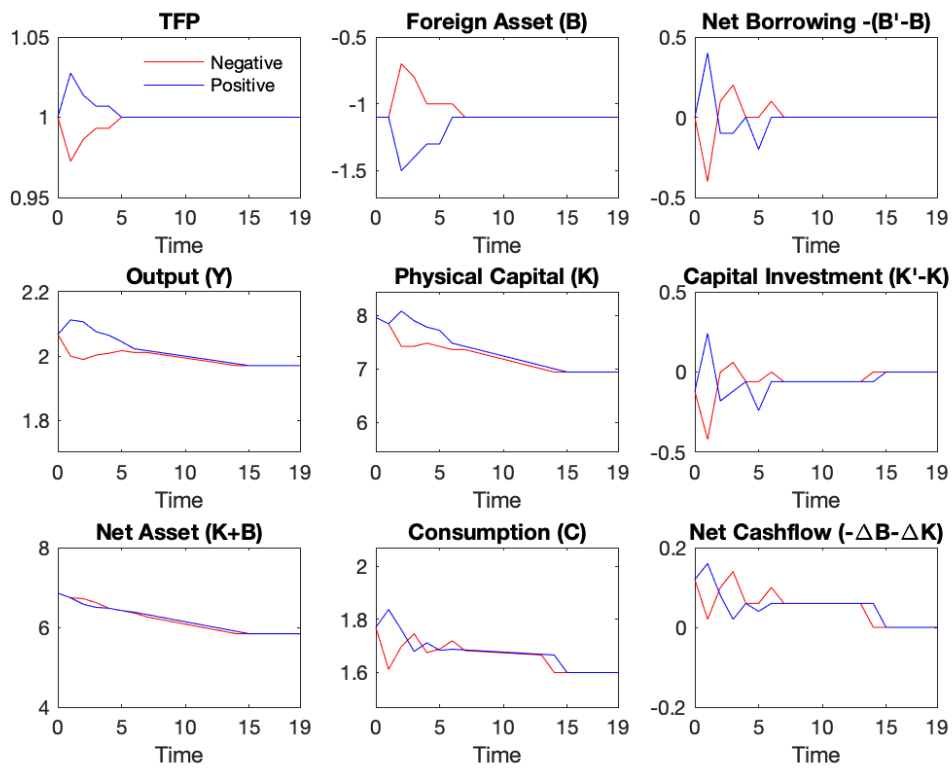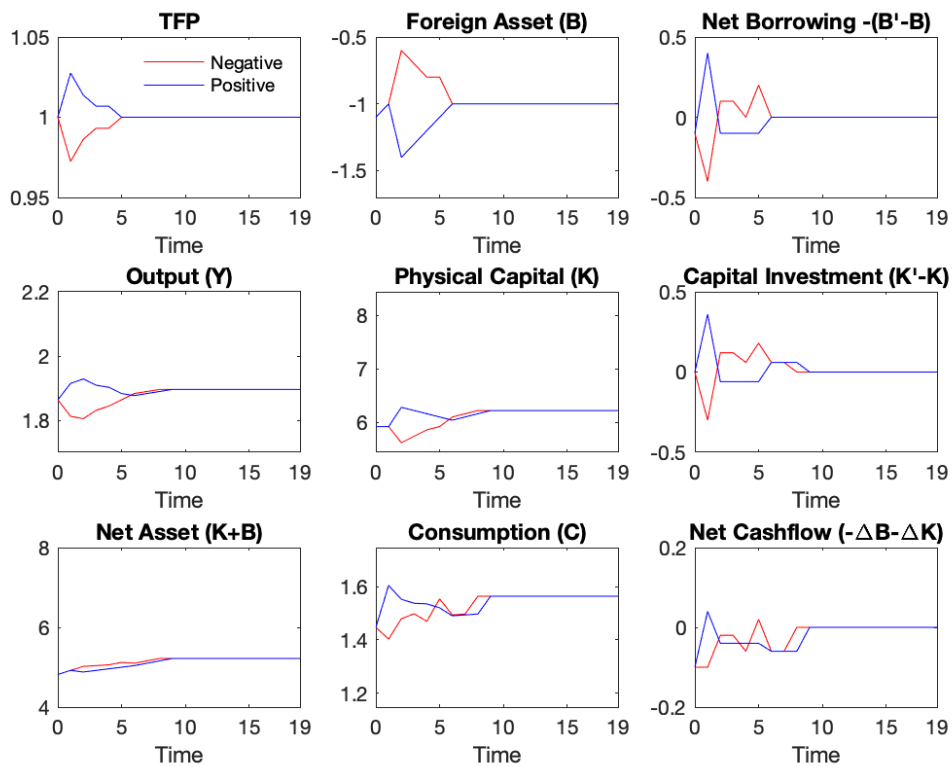

Figure 6: Impulse Response Functions at the steady state level of physical capital

(a) Impulse Response Functions at the above steady state level of capital

(b) Impulse Response Functions at the below steady state level of capital

Figure 7: Impulse Response Functions at the above and below steady state of capital

On the other hand, for a negative TFP shock in period 1, the country decides to repay with a sharp decrease in capital investment, while consumption is slightly reduced. Interestingly, Figure 6 shows consumption smoothing achieved by selling capital. This result can be explained by the fact that borrowing may not be better off because of the higher risk premium that will be face charged during a bad period (see Figure 2). As a result, the country can choose to smoothen consumption through selling physical capital.

Moreover, this section also provides the impulse response functions at above and below the steady state level of capital in Figure 7. As shown in Figure 7a, the country begins with an initial level of capital above the steady state. When a positive shock occurs in period 1, the country will choose to borrow more for both investment and consumption. The country can maintain a high level of debt with bond higher than the initial level, while a positive TFP shock still persists. However, after period 1, the country starts to sell physical capital and repay its debt. As can be seen in Figure 7a, net asset continuously declines until reaching its steady-state at period 15, as same as consumption. Comparing with the previous figure, consumption in Figure 7a can be maintained above the steady-state for 15 periods. This implies that a country with capital above the steady-state will sell assets for consumption.

Figure 7b shows opposite result for an initial capital below the steady state. With an increase in TFP in period 1, the country will start to borrow for both consumption and investment. As expected, the country accumulates net asset by borrowing and investing. However, as the initial capital is below the steady-state, the borrower has lower ability to borrow in comparison with the previous outcomes. Hence, physical capital slightly increases after the positive TFP shock, but it still remains below the

actual steady state.

Therefore, the impulse response functions analysis from 7 implies that a country will respond differently to TFP shocks, depending on initial the level of physical capital. If initial capital is above the steady state level, they can borrow more for both consumption and investment during a good period, and then sell capital to enjoy consumption above the steady-state. If initial capital is below the steady state, they will have a lower ability to borrow for investment. Physical capital is unable to return to its steady state.

Additionally, we also provide a simulation plot for a country starting with zero debt and various levels of physical capital (without shocks) in Figure 8.
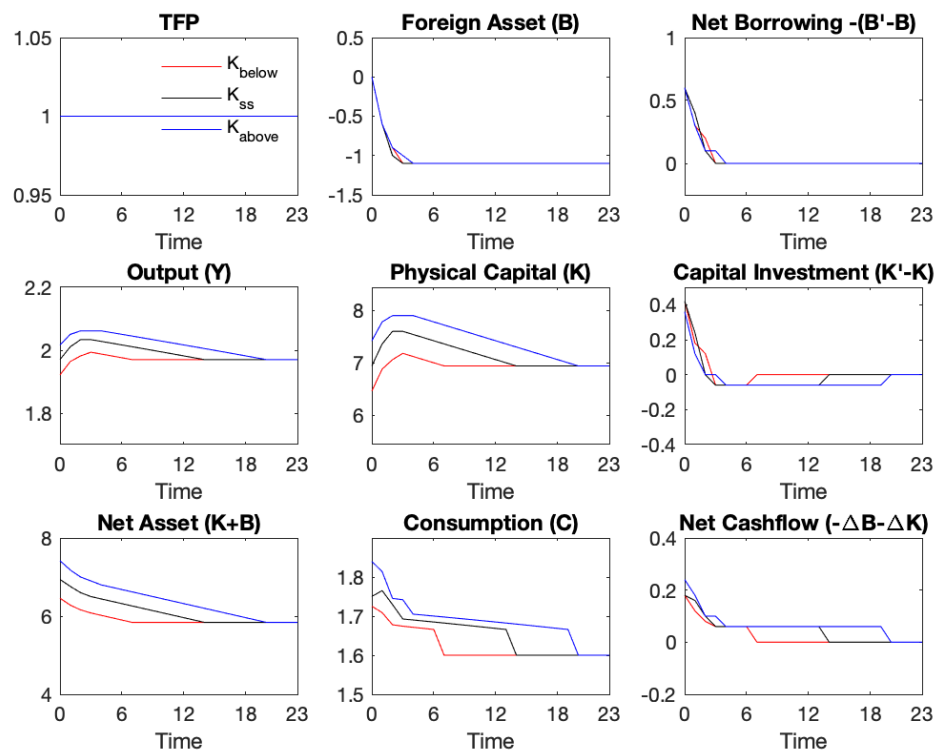

Figure 8: Impulse Response Functions at the steady state level of physical capital

As can be seen in the figure above, regardless of the initial levels of physical capital, a country will converge to the same steady-state of capital after 20 periods. A

country with initial debt below the optimal level will choose to borrow more.From the plot, it is obvious that the majority of its debt is used for investment. During the first three periods, capital investment is significantly higher than a change in consumption. After that, the country stops borrowing and smoothen consumption through selling capital until it reaches its steady-state. This outcome confirms that there is only one steady-state level of capital and debt. However, the country will not be able to invest and reach the steady-state level of capital, if it starts with less capital and high debt.

The Monte Carlo simulation over 1,000 periods is shown in Figure 9. All variables are plotted in real term.
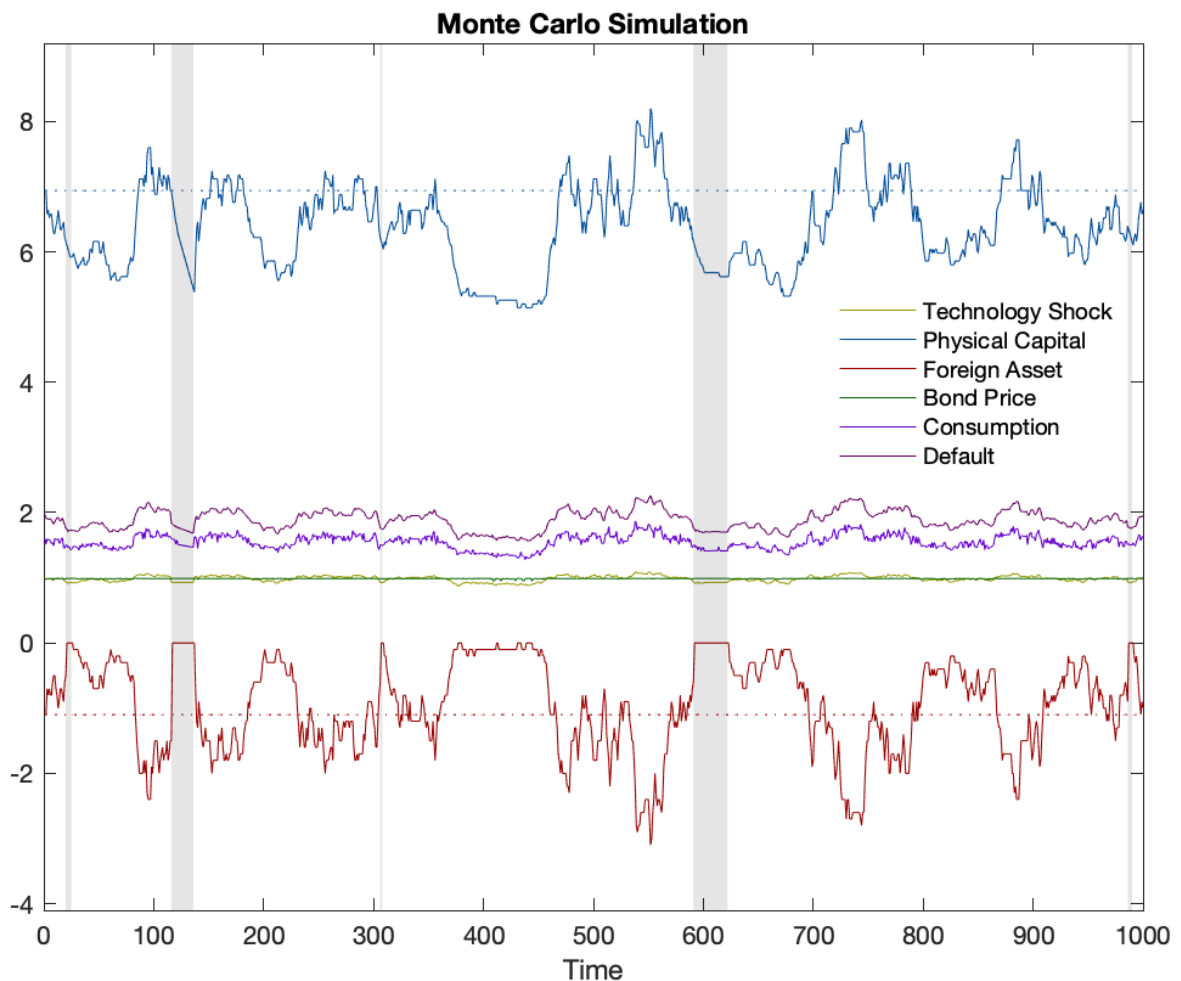

Figure 9: Monte Carlo Simulation over 1,000 periods

In the figure, the simulation starts at the economy steady-state and target the business cycle statistics of the Argentine economy, as shown in Table 4. The overlay default band corresponds to the sovereign default decision in the model; the dotted lines represent the steady-state level of capital and debt.

From Figure 9, we can see that the government tends to default after facing a series of negative shocks or a very large drop in output. After each default, the country will be excluded from the international bond market with a probability of re-entry (θ) equal to 5%. Output is capped at 7% below its mean during the exclusion periods. As can be seen in Figure 9, debt is always zero during defaults. Furthermore, after re-entering the bond market, the government will start borrowing and consuming more for a few periods until achieving that economy steady-state.

In addition, the government will choose to stay in the bond market with a repayment decision, if there is insufficient TFP shock or enough physical capital. During a slight drop in output, the sovereign debtor tends to sell physical capital or reduce consumption in order to repay its debt. Bond price fluctuates after a TFP shock with higher debt ratio. Interestingly, in some periods with a series of small negative shocks, the borrower sells physical capital to slowly repay its debt. With small debt and sufficient capital, the country can remain in the financial market during recession and wait for a positive shock.

The final and third part of this section will present the quantitative results of the Monte Carlo simulations over 100,000 periods for the Argentina's economy between 1980 Q1 and 2017 Q4.

Table 5: Model and Target Statistics

Parameters		Model	Target
Output volatility	σ_y	0.1009	0.0917
Consumption volatility	σ_c	0.0881	0.0969
Consumption output volatility	$\frac{\sigma_c}{\sigma_y}$	0.8733	1.0563
Correlation coefficient (c,y)	ρ_{cy}	0.9692	0.9223
Average consumption ratio	$\mu_{\frac{c}{y}}$	81.94 %	79.63 %
Debt ratio at steady state	μ_b	55.84 %	47.67 %
Length of default periods		4.18%	12.50%

As shown in Table 5, output volatility (σ_y), consumption volatility (σ_y) and

consumption output volatility $(\frac{\sigma_c}{\sigma_y})$ are 0.1009, 0.0881 and 0.8733 respectively. The

correlation coefficient between consumption and output (ρ_{cy}) is 0.9692. The results

are consistent with the Argentine data. More importantly, the model can target an

average consumption and debt ratio[7] of 81.94% and 55.84% respectively. However,

the length of the default period from the model simulation is 4.18 %, while the

actual data is 12.50 %.

Table 6: Other Business Cycle Statistics from the Model and the Actual Data

Business Cycle Statistics		Model	Actual
Debt Volatility	σ_b	0.4186	0.2719
Risk Premium Volatility	σ_ψ	0.0043	0.1420
Autocorrelation of Output	ρ_y	0.9909	0.6786
Autocorrelation of Consumption	ρ_c	0.9356	0.7860
Correlation coefficient (y,ψ)	$\rho_{y\psi}$	-0.2680	-0.4713
Correlation coefficient (c,ψ)	$\rho_{c\psi}$	-0.3070	-0.5402
Minimum Consumption Ratio	$\frac{c}{y}_{min}$	72.56 %	72.42 %
Maximum Consumption Ratio	$\frac{c}{y}_{max}$	97.22 %	90.39 %
Minimum Debt Ratio	b_{min}	0 %	9.07 %
Maximum Debt Ratio	b_{max}	194.05 %	153.63 %

Finally, the other statistic from the model and the actual data is shown in Table

6. Volatilities of debt and risk premium are 0.4186 and 0.0043 respectively. The

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[7]The model uses debt ratio at the steady state in order to target the average debt ratio of
Argentina between 1980 Q1 and 2017 Q4.

autocorrelation of output and consumption are 0.9909 and 0.9356. Besides, the model shows a negative relationship between output and risk premium of -0.2680, whilst the relationship between consumption and risk premium is -0.3070. Minimum and maximum values of the of consumption and debt ratios are 72.56%, 97.22%, 0% and 194.05% respectively. The correlation coefficient that ensure the policy function among output and risk premium matches actual data. However, risk premium in the model is less volatile, because the sovereign debtor will choose to default when borrowing is expensive. As a result, in comparison of the previous literature, the statistical results of this paper can provide a closer fit to the actual data.

6 Conclusion

This paper contributes to the literature on sovereign debt default by modelling incomplete information about borrower's physical capital. We find that incomplete information about a borrower's physical capital affects the borrower's default decision and their optimal capital, consumption and debt.

In our model, the default decision by a borrower will be punished with their financial exclusion and by them bearing the cost of default. These two punishments will result in lower utility in future periods. Equilibrium bond price will be higher than predicted by previous literature as the lender cannot observe the borrower's capital and therefore in equilibrium a higher risk premium emerges. Without productivity shocks the steady-state equilibrium will be characterised by no default occurring. On the other hand, productivity shocks will affect capital, consumption and debt and may lead to the default decision.

Furthermore, we find that borrowers choose to borrow more when it is possible. In comparison with previous literature, our model simulations indicate a higher level of debt (in and out of the steady state) which fits to the actual data for Argentina.

Unobservable capital implies a higher risk premium in equilibrium. In equilibrium, bond price will not be directly affected by a change in future capital investment. The risk premium and bond price emerge when the actual output is revealed. The borrowers cannot change the bond price schedule by changing their future capital investment.

Another novel result is that the steady-state level of physical capital is typically less than in the previous literature because borrowers have no incentive to accumulate capital for influencing the bond price in the current period.

This framework also suggests a smoothing of physical capital when shocks occur, which is not captured by previous literature, as changes in future capital investment have no direct impact on the current bond price schedule.

The results of impulse response functions suggest that it is not optimal to force debtors to use foreign assets to invest only. The borrower will find optimal to consume rather than invest, if physical capital is above the steady-state level. On the other hand, at or below the steady-state level of capital, the country will borrow more for both consumption and investment in the presence of a positive shock.

Our simulations with physical capital at or above steady-state level also show consumption smoothing behaviour through sacrificing physical capital during bad periods. This, consistently with previous literature, is because consumers will take decisions on the amount of capital investment and debt in order to keep

consumption at an optimal level. However, when large negative productivity shock occurs, we find that borrowers are better off if they reduce consumption and choose to repay their debt in order to avoid default.

The simulation results show that our model is better suited to fitting Argentina's default experience than existing models in the literature and it confirms the importance of limited information about a borrower's physical capital for understanding the behaviour of international borrowers. Therefore, this paper provides a seminal framework for further studies on sovereign default. An interesting extension would be to model borrowers and lenders with different features; for example, different risk aversion rates and discount rates and the possibility of partial default.

7   Appendix

This appendix will be separated into two main parts. The first part will describe the data sources. The second part will describe the computational algorithm on the model of sovereign default and productivity shock with the finite state Markov-Chain approximation (Tauchen, 1986; Tauchen and Hussey, 1991).

7.1   Data Description

The data series are taken from the Ministry of Economy (MECON) of Argentina and complied by Datastream. These include real GDP, household consumption, government spending that present in real term and external debt as a percentage to GDP. Besides, The overall risk premium on US denominated debt is from JP Morgan

and compiled by Oxford Economics. All data are quarterly series with non-seasonal
adjustment between 1980 Q1 and 2017 Q4.

7.2 Computational Algorithm

In this thesis, we discretise a continuous stochastic process of TFP by using
Tauchen's method (1986). The TFP level is defined as follows:

$$a_t = \rho \, a_{t-1} + \epsilon_t \,, \tag{7.1}$$

where $|\rho| < 1$ and $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. From this continuous stochastic process, we will
assume $\tilde{a}$ as the discrete value from the approximation of a with the finite set of
possible realisation $\{a_1, a_2, ... a_N\}$. Tauchen (1986) also suggests that a maximum
value of $\tilde{a}$ (a_N) is given by:

$$a_N = m \left(\frac{\sigma_\epsilon^2}{1 - \rho^2} \right)^{\frac{1}{2}} \,, \tag{7.2}$$

where m is a multiple of the unconditional standard deviation. In general case, m is
equal to 3. From the symmetric assumption of the distribution, the minimum value
of $\tilde{a}$ (a_1) is $-a_N$. Besides, $\{a2, a3, ..., a_{N-1}\}$ will be equivalently located between
the interval $[a_1, a_N]$ with the distance (d). Therefore, the finite state of possible
realisation of $\tilde{a}$ can be obtained as the state space form.

Next, Tauchen (1986) provides a solution to compute the transition probabilities as
follows:

$$\pi_{jk} = P\left\{\tilde{a}_t = a_k | \tilde{a}_{t-1} = a_j\right\} = P\left\{a_k - \frac{d}{2} - \rho a_j < \epsilon_t \le a_k + \frac{d}{2} - \rho a_j\right\}. \tag{7.3}$$

If $1 < k < N - 1$, the transition probability π_{jk} will be given by:

$$\pi_{jk} = F\left(\frac{a_k + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right) - F\left(\frac{a_k - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right), \qquad (7.4)$$

where F is the standardisation process. When $k = 1$ and $k = N$ the transition probability at these boundaries are given by:

$$\pi_{j1} = F\left(\frac{a_1 + \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right), \qquad (7.5)$$

$$\pi_{jN} = 1 - F\left(\frac{a_N - \frac{d}{2} - \rho a_j}{\sigma_\epsilon}\right). \qquad (7.6)$$

From the above method, the possible set of $\tilde{a}$ with its transition probability will converge to a_t when the distance (d) is approaching 0 and N is closed to ∞. Therefore, a is defined by the finite vector of possible realisation of TFP with the interval $[\underline{a}, \overline{a}]$, whilst the continuous stochastic process of TFP is approximated by the given transition probability matrix.

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