Optimal Environmental Taxation in the Presence of Pollution and Congestion: A General-Equilibrium Analysis

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Abstract

We derive the structure of optimal fuel and road taxs in an economy with (i) vintage vehicles (new and old vehicles) and, (ii) pollution and congestion externalities caused by driving. It is shown that the optimal road taxes on new vehicles are higher in that new vehicles generate more mileage of travel which implies more congestion. We also derive the optimal fuel tax when it needs to be uniform. We next calibrate the model on U.S. data. Here, in the presence of a congestion externality, the optimal fuel tax for old vehicles is higher which shows that the marginal cost of pollution outweighs the marginal cost of congestion. When we implement a uniform fuel tax, its rate lies between optimal ones for new and old vehicles and the road tax is the same for both types of vehicles. We also find that long-run utility is marginally higher under a full set of tax instruments.

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1 Introduction

Fuel consumed during driving creates externalities through pollution, congestion, accidents and import dependence (Haughton and Sarkar, 1996). How to guarantee efficiency of a competitive process and address externalities have been an important problem of constructing economic policy. Environmental taxes, internalizing the external costs that fuel consumption imposes on the rest of the society, have been a popular policy tool to address externalities, especially pollution and congestion (Bovenberg and De Mooij, 1994). However what we observe is that fuels are taxed at widely different rates in different countries (Newbery, 2005), with UK in particular stands out as having high oil taxes and USA specifically low in its oil taxes among all the OECD countries (OECD, 2018). One is naturally promoted to ask whether environmental taxes chosen by different countries are appropriate or not.

We focus on two important externalities generated by fuel via driving. The first external impact is pollution which is viewed as a byproduct of gasoline combustion during driving. The emissions of carbon dioxide, nitrogen oxides and monoxide still pose great threats especially to urban areas. The latter two are the main cause for smog while carbon dioxide accumulates and contributes to greenhouse effect which might contributes to global warming (Haughton and Sarkar, 1996). The second externality caused by driving is congestion. Gasoline is mainly used in motor vehicles (Haughton and Sarkar, 1996) and the more often households drive their vehicles, the heavier the traffic is.

A number of previous empirical studies attempt different ways to quantify the external costs generated[1] and most of their estimation is mileage-based. Parry and Small (2005) builds up a static analytical framework and solve for second-best optimal fuel tax and disaggregates it into components that reflect different external costs. They then calibrate their model based on the UK economy and US economy and explain why different countries have different fuel tax rates. However, there are still limitations within the previous research: First, congestion itself cannot be fully addressed by only taxing fuel. Congestion is normally measured by the time spent on road. Driving time is determined as the inverse of average travel speed times mileage of travel (Parry and Small, 2005). As agents normally take speed as given, the higher mileage of travel, the longer time agents have to spend on road which means heavier traffic. Mileage of travel is produced by different fuel-efficiency-level of cars and gasoline. Therefore, simply by charging higher price on fuel would not fully solve congestion externality. Second, it is crucial to take into consideration of the endogeniety of fuel efficiency. As fuel becomes more expensive, households respond to this by either driving more fuel-efficient vehicles or driving less, which means that fuel economy of the vehicle fleet matter. Third, fuel efficiency progresses over time and more fuel-efficient vehicles contributes less emissions thus lower pollution level. To capture the long-run impact of policy on environment, the contribution of

[1] For example Peirson et al. (1995), Mayeres et al. (1996) and Rothengatter and Mauch (2000)

a static model is very much diminished.

This paper contributes to the theoretical literature in several ways. First, we examine the first best environmental taxes to address pollution and congestion externalities separately. We also show how fuel tax and road tax interact with each other. Second, we introduce capital heterogeneity using ”putty-putty” technology (see Cooley et al. (1997) and Solow et al. (1960)) to model vehicles of different vintage so as to better capture the impact of fuel efficiency endogeneity on optimal environmental taxes. Third, a dynamic view is useful in interpreting pollution externalities as emissions accumulate overtime and impact agents in the long-run. This paper examines the first best optimal environmental taxes (fuel tax and road tax) employing a two-period vintage dynamic general equilibrium model with pollution and congestion externalities presented.

We summarize the results as follows. First, analytical results show that the first best optimal fuel tax consists of two parts: marginal cost of pollution and marginal cost of congestion. New cars generates less pollution but contribute more to mileage of travel which leads to more congestion. Thus the optimal fuel tax of different types of vehicles depends on these two contradicting powers. Optimal road tax targets at the congestion externality which is related to vehicle fuel efficiency level. In steady states, households prefer to drive new cars more often which implies higher mileage of travel, thus road tax is higher for new cars than old cars. We further solve for uniform fuel tax and it takes the form of weighted average of fuel taxes of new cars and old cars. Second, we calibrate our model based on US economy and show that optimal environmental taxes depend on preference parameters. In the presence of congestion externality, optimal fuel tax for old vehicles is higher when households start to value environment which shows that the marginal cost of pollution outweighs the marginal cost of congestion. Households are better off under optimal fuel tax than uniform fuel tax but not to a substantial extent.

The paper is organized as follows. Section 2 describes the model. Section 3 solves the social planner's problem and describes its dynamics while section 4 looks at the decentralized economy case. Section 5 and 6 presents the environmental taxes solutions (fuel tax and road tax). Section 7 describes calibration and numerically present the environmental taxes under different sets of preference parameters. Section 8 concludes.

2 Model Setting

2.1 Driving Behavior

The driving service is provided by both new and old cars:

$$M_t = (m_{t,1}^{\sigma} + m_{t,2}^{\sigma})^{\frac{1}{\sigma}}, \qquad 0 < \sigma < 1 \tag{1}$$

At each time period, the mileage provided by the new cars and the old cars is:

$$m_{t,1} = (a_{t-1}\delta_{t-1})^{\gamma} g_{t,1} \tag{2}$$

$$m_{t,2} = (\rho a_{t-2}\delta_{t-2})^{\gamma} g_{t,2}, \qquad 0 < \rho < 1 \tag{3}$$

2.2 Environment

We assume that the average speed of people driving is an exogenous constant. Thus, we could use the sum of mileage to proxy for congestion externality.

$$N_t = m_{t,1} + m_{t,2} \tag{4}$$

Household gains utility from good environment quality. However, gasoline combustion caused by driving will cause pollution which will also be mitigated by more fuel-efficient vehicles. Moreover, each period, nature will absorb certain amount of pollutants and improve environment quality.

Pollution is caused by the usage of gasoline but mitigated by fuel-efficient vehicles:

$$P_t = \frac{g_{t,1}}{\delta_{t-1}} + \frac{g_{t,2}}{\rho\delta_{t-2}} \tag{5}$$

Environment quality will be improved each period by nature's absorbing pollutants ability. Moreover, environment quality will not explode, thus there will be an upper limit for it.

$$Q_{t+1} - Q_t = \Phi - \epsilon Q_t - P_t, \qquad Q_{max} = \bar{Q} \tag{6}$$

3 Social Planner's Problem

In this section, we solve the problem where social planner allocates the resources:

$$\begin{aligned} &V^t(k_t, Q_t; a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}; \{I_t\}) = \\ &\max_{c_t, g_{t,1}, g_{t,2}, a_t, l_t^g, l_t, k_t^g, k_t} [U(c_t, M_t, 1-l_t, N_t, Q_t) + \beta V^{t+1}(k_{t+1}, Q_{t+1}; a_t\delta_t, \rho a_{t-1}\delta_{t-1}; \{I_{t+1}\})] \end{aligned} \tag{7}$$

subject to:

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1-\epsilon_k)k_t + p_t(g_{t,1} + g_{t,2})$$

where $k_t = k_t^a + k_t^g$ and $l_t = l_t^a + l_t^g$.

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t$$

and

$$Q_{t+1} - Q_t = \Phi - \epsilon Q_t - P_t \qquad\qquad Q_t \leq \bar{Q}$$

The resource constraint and the equilibrium condition imply that:

$$\delta_t = H(k_t - k_t^g, l_t - l_t^g, a_t)$$

Thus we could obtain F.O.C:

$$c_t: \quad U_c = \beta V_{k_{t+1}}^{t+1} \tag{8}$$

$$g_{t,1}: \quad U_{M_t} \frac{\partial M_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t} \frac{\partial N_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial g_{t,1}} = \beta \left(p_t V_{k_{t+1}}^{t+1} + V_{Q_{t+1}}^{t+1} \frac{\partial P_t}{\partial g_{t,1}} \right) \tag{9}$$

$$g_{t,2}: \quad U_{M_t} \frac{\partial M_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial g_{t,2}} + U_{N_t} \frac{\partial N_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta \left(p_t V_{k_{t+1}}^{t+1} + V_{Q_{t+1}}^{t+1} \frac{\partial P_t}{\partial g_{t,2}} \right) \tag{10}$$

$$a_t: \quad \beta V_{a_t \delta_t}^{t+1} \left(\delta_t + a_t \frac{\partial H}{\partial a_t} \right) = 0 \tag{11}$$

$$l_t^g: \quad \beta \left[V_{k_{t+1}}^{t+1} \frac{\partial G}{\partial l_t^g} - V_{a_t \delta_t}^{t+1} a_t \frac{\partial H}{\partial (l_t - l_t^g)} \right] = 0 \tag{12}$$

$$l_t: \quad U_{1-l_t} = \beta V_{a_t \delta_t}^{t+1} a_t \frac{\partial H}{\partial (l_t - l_t^g)} \tag{13}$$

$$k_t^g: \quad V_{k_{t+1}}^{t+1} \left(\frac{\partial G}{\partial k_t^g} \right) - V_{a_t \delta_t}^{t+1} \left[a_t \frac{\partial H}{\partial (k_t - k_t^g)} \right] = 0 \tag{14}$$

Envelop conditions:

$$V_{k_t}^t = \beta V_{k_{t+1}}^{t+1} (1 - \epsilon_k) + \beta V_{a_t \delta_t}^{t+1} a_t \frac{\partial H}{\partial (k_t - k_t^g)} \tag{15}$$

$$V_{Q_t}^t = U_Q + \beta V_{Q_{t+1}}^{t+1} (1 + \epsilon) \tag{16}$$

$$V_{(a_{t-1}\delta_{t-1})}^t = U_{M_t} \frac{\partial M_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial (a_{t-1}\delta_{t-1})} + U_{N_t} \frac{\partial N_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial (a_{t-1}\delta_{t-1})} + \rho \beta V_{(\rho a_{t-1}\delta_{t-1})}^{t+1} \tag{17}$$

$$V_{(\rho a_{t-2}\delta_{t-2})}^t = U_{M_t} \frac{\partial M_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial (\rho a_{t-2}\delta_{t-2})} + U_{N_t} \frac{\partial N_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial (\rho a_{t-2}\delta_{t-2})} \tag{18}$$

4 Decentralized Economy

Now we start to look at the scenario where we have many firms and many households. Households own all factors of production and all shares in firms. We also have government in the economy and it collects tax from consumption of gasoline and use the revenue to subsidize the production of more efficient vehicles for the second type of firms.

4.1 Firms

We have two types of firms: one is for production of general consumption goods, accumulation of capital and also purchase gasoline. The second type of firms will produce vehicle capital and fuel efficiency.

4.1.1 General Production

In this sector, firms hire labours l_t^g and rent capital k_t^g from households to produce consumption goods, accumulation of capital and import gasoline with constant-return-to-scale technology. The profits generated will go back to households.

$$G(k_t^g, l_t^g) = c_t + k_{t+1} - (1 - \epsilon_k)k_t + p_t(g_{t,1} + g_{t,2})$$

Thus the problem facing the firms in this sector is to maximize its profit:

$$\max_{k_t^g, l_t^g} \pi_t^g = G(k_t^g, l_t^g) - r_t^g k_t^g - w_t^g l_t^g$$

we normalize the price from general production to unity. Given its constant-return-to-scale technology, the profit from general production sector π_t^g will be zero.

4.1.2 Vehicle Production

In this sector, firms hire labours l_t^a and rent capital k_t^a to produce vehicle capital a_t and fuel efficiency δ_t. The firms sell the combination of vehicle capital and fuel efficiency to households with the price q_t^a. The firms in this sector will also receive subsidy s_t from government for producing more fuel-efficient vehicles.

$$F(k_t^a, l_t^a) = a_t + \mu\delta_t$$

Firm's goal is to maximise its profits:

$$\max_{k_t^a, l_t^a, \delta_t} \pi_t^a = q_t^a a_t \delta_t - r_t^a k_t^a - w_t^a l_t^a + s_t \delta_t$$

4.1.3 Equilibrium Conditions in Production

$$k_t^a + k_t^g = k_t \qquad l_t^g + l_t^a = l_t$$

$$w_t^a = w_t^g = w_t \qquad r_t^a = r_t^g = r_t$$

4.2 Households

Households gain utility from general consumption goods, driving service, leisure and environment quality. They get disutility from congestion N_t.

$$U(c_t, M_t, 1-l_t, N_t, Q_t)$$

where $U_c > 0$, $U_M > 0$, $U_l < 0$, $U_N < 0$ and $U_{Q_t} > 0$.We assume log-preferences for consumption, driving service, leisure, congestion and environmental quality.

$$U(c_t, M_t, 1-l_t, N_t) = \phi_1 \log c_t + \phi_2 \log M_t + (1-\phi_1-\phi_2)\log(1-l_t) + \phi_3 \log(\bar{N} - N_t) + \phi_4 \log Q_t \tag{19}$$

where ϕ_1, ϕ_2, ϕ_4 are positive while ϕ_3 is negative.

Every time period, households will supply labour and capital to firms and receive all the profits earned by two types of firms. Households will spend on consumption goods, gasoline, new vehicles and investment.

Thus the budget constraint facing households is:

$$\pi_t^a + \pi_t^g + w_t l_t + r_t k_t = (p_t + \tau_t^1) g_{t,1} + (p_t + \tau_t^2) g_{t,2} + k_{t+1} - (1-\epsilon_k) k_t + c_t + q_t^a (a_t \delta_t) + T_1(a_{t-1}\delta_{t-1}) + T_2(a_{t-2}\delta_{t-2}) \tag{20}$$

The problem household is facing is:

$$\begin{aligned} V^t(k_t, a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}; \{I_t, Q_t\}) &= \\ \max_{c_t, g_{t,1}, g_{t,2}, a_t\delta_t, l_t} [U(c_t, M_t, 1-l_t; \{N_t, Q_t\}) + \beta V^{t+1}(k_{t+1}, a_t\delta_t, \rho a_{t-1}\delta_{t-1}; \{I_{t+1}, Q_{t+1}\})] \end{aligned} \tag{21}$$

subject to the budget constraint shown in 20. Note that when making decisions, household will not internalize the detrimental effects caused by driving. Put differently, households will not consider externalities.

The first-order conditions are:

$$c_t: \quad U_c = \beta V_{k_{t+1}}^{t+1} \tag{22}$$

$$g_{t,1}: \quad U_{M_t} \frac{\partial M_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial g_{t,1}} = \beta V_{k_{t+1}}^{t+1} (p_t + \tau_t^1) \tag{23}$$

$$g_{t,2}: \quad U_{M_t} \frac{\partial M_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta V_{k_{t+1}}^{t+1} (p_t + \tau_t^2) \tag{24}$$

$$a_t\delta_t: \quad q_t^a V_{k_{t+1}}^{t+1} = V_{a_t\delta_t}^{t+1} \tag{25}$$

$$l_t: \quad U_{1-l_t} = -w_t \beta V_{k_{t+1}}^{t+1} \tag{26}$$

Similarly, we could get envelop conditions:

$$V^t_{k_t} = \beta(1-\epsilon_k + r_t)V^{t+1}_{k_{t+1}} \tag{27}$$

$$V_{Q_t} = U_{Q_t} + \beta(1+\epsilon)V^{t+1}_{Q_{t+1}} \tag{28}$$

$$V^t_{(a_{t-1}\delta_{t-1})} = U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + \rho\beta V^{t+1}_{(\rho a_{t-1}\delta_{t-1})} - \beta V^{t+1}_{k_{t+1}}T_1 \tag{29}$$

$$V^t_{(\rho a_{t-2}\delta_{t-2})} = U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} - \beta V^{t+1}_{k_{t+1}}T_2 \tag{30}$$

4.3 Government

Government levies tax on household's purchase of gasoline and uses the tax revenue to subsidize the production of more fuel-efficient vehicles in the vehicle production sector.

$$s_t\delta_t = \tau_t(g_{t,1} + g_{t,2}) \tag{31}$$

5 Optimal Environmental Taxation: First Best Case

5.1 Optimal Gasoline Tax

Taxes are used to correctly "price" social activities causing externalities, i.e. pollution and congestion. Gasoline taxes help prices closely approximate marginal social cost, that is, the gasoline tax household has to pay should equal exactly to the marginal social cost caused by gasoline consumption so as to achieve first best. Given that we have different types of vehicles, different and specific gasoline taxes need to be applied. Thus, using 9, 10, 23 and 24, we are able to equalize the marginal social cost and the tax.

9 and 23 render the optimal gasoline tax rate for new cars:

$$\tau^1_t = \frac{V^{t+1}_{Q_{t+1}}}{V^{t+1}_{k_{t+1}}}\frac{\partial P_t}{\partial g_{t,1}} - \frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}}{\beta V^{t+1}_{k_{t+1}}} \tag{32}$$

Similarly, 10 and 24 give us the optimal gasoline tax rate for old cars:

$$\tau^2_t = \frac{V^{t+1}_{Q_{t+1}}}{V^{t+1}_{k_{t+1}}}\frac{\partial P_t}{\partial g_{t,2}} - \frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}}{\beta V^{t+1}_{k_{t+1}}} \tag{33}$$

Gasoline consumption is related to both types of externalities: pollution and gasoline consumption, which is priced accordingly in Eq.32 and 33.

We are also interested to see which gasoline tax is higher, in steady state, $\tau^2 - \tau^1$ become:

$$\tau^2 - \tau^1 = \frac{\overset{+}{V_Q}}{V_k}(\overset{+}{\frac{1}{\rho\delta}} - \frac{1}{\delta}) + \frac{\overset{-}{U_N}}{\beta V_k}[(a\delta)^\gamma \overset{+}{-} (\rho a\delta)^\gamma] \tag{34}$$

Thus, in steady state, the magnitude of gasoline tax is undetermined analytically. It depends on the contradicting power between marginal cost of pollution and marginal cost of congestion caused by gasoline consumption. However, it is clear that for the pollution externality caused by gasoline consumption, old cars need to be taxed more while for the congestion externality caused by gasoline consumption, new cars need to be taxed more.

5.2 Optimal Road Tax

Road taxes are used to correct congestion externalities. Compare the value functions from social planner's problem and decentralized economy, we obtain:

$$U_{N_t} \frac{\partial N_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} = -\beta V_{k_{t+1}}^{t+1} T_1 \tag{35}$$

and

$$U_{N_t} \frac{\partial N_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} = -\beta V_{k_{t+1}}^{t+1} T_2 \tag{36}$$

which render the solutions to optimal road tax:

$$T_1 = -\frac{U_{N_t} \frac{\partial N_t}{\partial m_{t,1}} \frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}}{\beta V_{k_{t+1}}^{t+1}} \tag{37}$$

and

$$T_2 = -\frac{U_{N_t} \frac{\partial N_t}{\partial m_{t,2}} \frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}}{\beta V_{k_{t+1}}^{t+1}} \tag{38}$$

Similarly, in steady state, we want to see the comparison between marginal congestion cost of new cars and old cars:

$$T_1 - T_2 = \frac{U_N}{\beta V_k}\gamma(a\delta)^{\gamma-1}(\rho^{\gamma-1} g_2 - g_1) \tag{39}$$

which again is undetermined and depends on the gasoline consumption ratio between new cars and old cars.

6 Uniform Environmental Tax

Levying different tax rates based on the type of vehicles is difficult in terms of practicality. Thus, we are intrigued to find the uniform environmental tax rates in second best.

6.1 Optimal Uniform Gasoline Tax

To have uniform gasoline tax, left hand side of 23 and 24 must forced to be the same.

Thus, we have:

$$U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} = U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{m_{t,2}}{\partial g_{t,2}}$$

which will give us the condition on gasoline consumption ratio:

$$g_{t,1} = (\frac{a_{t-1}\delta_{t-1}}{\rho a_{t-2}\delta_{t-2}})^{\frac{\gamma\sigma}{1-\sigma}} g_{t,2} = (\frac{\rho a_{t-2}\delta_{t-2}}{a_{t-1}\delta_{t-1}})^{\frac{\gamma\sigma}{\sigma-1}} g_{t,2} \tag{40}$$

and it can be expressed using general form:

$$g_{t,1} = \Phi(a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}) g_{t,2} \tag{41}$$

Putting this in to social planner's problem, $g_{t,1}$ is not going to be a choice problem now for the planner, equation 9 disappear. Instead, considering 41,equation 10 becomes:

$$\begin{aligned} &U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} - \beta(p_t V^{t+1}_{k_{t+1}} + V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,2}}) + \\ &\left[U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} - \beta(p_t V^{t+1}_{k_{t+1}} + V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}})\right]\Phi = 0 \end{aligned} \tag{42}$$

All the other first-order conditions will remain the same. The change in $a_{t-1}\delta_{t-1}$ and $\rho a_{t-2}\delta_{t-2}$ will also affect the change in g_1. Thus, envelop conditions 17 and 18 will change into:

$$\begin{aligned} &V^t_{(a_{t-1}\delta_{t-1})} = U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial (a_{t-1}\delta_{t-1})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial (a_{t-1}\delta_{t-1})} + \rho\beta V^{t+1}_{(\rho a_{t-1}\delta_{t-1})} \\ &\left[U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} - \beta(p_t V^{t+1}_{k_{t+1}} + V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}})\right]\frac{\partial g_{t,1}}{\partial (a_{t-1}\delta_{t-1})} \end{aligned} \tag{43}$$

$$\begin{aligned} &V^t_{(\rho a_{t-2}\delta_{t-2})} = U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial (\rho a_{t-2}\delta_{t-2})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial (\rho a_{t-2}\delta_{t-2})} \\ &\left[U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}} - \beta(p_t V^{t+1}_{k_{t+1}} + V^{t+1}_{Q_{t+1}}\frac{\partial P_t}{\partial g_{t,1}})\right]\frac{\partial g_{t,1}}{\partial (\rho a_{t-2}\delta_{t-2})} \end{aligned} \tag{44}$$

Rearrange 42 and multiply each side by g_2:

$$\left(U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}+U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}\right)g_{t,2}+\left(U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\right)g_{t,1}$$
$$=\beta\left[p_tV_{k_{t+1}}^{t+1}(g_{t,1}+g_{t,2})+V_{Q_{t+1}}^{t+1}(\frac{\partial P_t}{\partial g_{t,2}}g_{t,2}+\frac{\partial P_t}{\partial g_{t,1}}g_{t,1})\right]$$

We show that the value functions under the gasoline consumption constraint match with the value functions for social planner. We form a constrained social planner's problem in Appendix **??**.

Now we can solve the uniform tax under gasoline consumption ratio constraint. For households in decentralized economy, they still make decisions separately on the consumption of gasoline (g_1 and g_2). However, they are now facing a uniform tax τ_t on gasoline in stead of separate ones.

Thus 23 and 24 are changed into:

$$U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}=\beta V_{k_{t+1}}^{t+1}(p_t+\tau_t) \tag{45}$$

$$U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}=\beta V_{k_{t+1}}^{t+1}(p_t+\tau_t) \tag{46}$$

Substitute into 42, we could get:

$$\beta V_{k_{t+1}}^{t+1}(p_t+\tau_t)g_{t,2}+U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}}g_{t,2}-\beta\left(p_tV_{k_{t+1}}^{t+1}+V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,2}}\right)g_{t,2}$$
$$+\beta V_{k_{t+1}}^{t+1}(p_t+\tau_t)g_{t,1}+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}g_{t,1}-\beta\left(p_tV_{k_{t+1}}^{t+1}+V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\right)g_{t,1}=0$$

which renders the solution for uniform tax rate:

$$\tau_t=\frac{g_{t,2}}{g_{t,1}+g_{t,2}}\tau_t^2+\frac{g_{t,1}}{g_{t,1}+g_{t,2}}\tau_t^1 \tag{47}$$

The uniform tax rate is the weighted average of the separate ones.

6.2 Optimal Road Tax under Constraint

The gasoline consumption condition also affects road tax. We compare Eq.29 with Eq.43, and Eq.30 with Eq.44:

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}+\left[\beta V_{k_{t+1}}^{t+1}\tau_t+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}-\beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\right]\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}=-\beta V_{k_{t+1}}^{t+1}T_1^c \tag{48}$$

Similarly,compare 18 and 30, we get:

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}+\left[\beta V_{k_{t+1}}^{t+1}\tau_t+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}-\beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\right]\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}=-\beta V_{k_{t+1}}^{t+1}T_2^c \tag{49}$$

where T_1^c and T_2^c denote road taxes under constrained condition for new cars and old cars.

Eq.32 means that:

$$\beta V_{k_{t+1}}^{t+1}\tau_t=\beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}-U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}$$

Substitute into the two expressions above:

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}+\beta V_{k_{t+1}}^{t+1}(\tau_t-\tau_t^1)\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}=-\beta V_{k_{t+1}}^{t+1}T_1$$

$$U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}+\beta V_{k_{t+1}}^{t+1}(\tau_t-\tau_t^1)\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}=-\beta V_{k_{t+1}}^{t+1}T_2$$

Divide $-\beta V_{k_{t+1}}^{t+1}$ on both sides of the equations:

$$T_1^c=-\frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}}{\beta V_{k_{t+1}}^{t+1}}-(\tau_t-\tau_t^1)\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}$$

$$T_2^c=-\frac{U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})}}{\beta V_{k_{t+1}}^{t+1}}-(\tau_t-\tau_t^1)\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}$$

using Eq. 37 and Eq.38:

$$T_1^c=T_1-(\tau_t-\tau_t^1)\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} \tag{50}$$

$$T_2^c=T_2-(\tau_t-\tau_t^1)\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} \tag{51}$$

Given Eq.47:

$$\tau_t-\tau_t^1=\frac{g_{t,2}}{g_{t,1}+g_{t,2}}(\tau_t^2-\tau_t^1)$$

Using 32 and 33:

$$\tau_t^2-\tau_t^1=\frac{V_{Q_{t+1}}}{V_{k_{t+1}}^{t+1}}\left(\frac{1}{\rho\delta_{t-2}}-\frac{1}{\delta_{t-1}}\right)+\frac{U_{N_t}}{\beta V_{k_{t+1}}^{t+1}}[(a_{t-1}\delta_{t-1})^\gamma-(\rho a_{t-2}\delta_{t-2})^\gamma]$$

In steady state, $\tau^2-\tau^1$ become:

$$\tau^2 - \tau^1 = \frac{\overset{+}{V_Q}}{V_k}(\overset{+}{\frac{1}{\rho\delta} - \frac{1}{\delta}}) + \frac{\overset{-}{U_N}}{\beta V_k}[\overset{+}{(a\delta)^\gamma - (\rho a\delta)^\gamma}] \quad (52)$$

Thus the sign of $\tau - \tau^1$ can not determined in steady state.

From 40, we get:

$$\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} > 0 \qquad \frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} < 0$$

Thus it still remains unknown analytically whether the road tax with constraint is bigger or smaller than the ones without. To have a better picture of the tax rates and their interactions among each other under the gasoline consumption constraint, numerical simulation is needed.

7 Numerical Solutions to Optimal Gasoline and Road Tax

In this section, we employ a numerical model of the U.S. economy to examine the first- and second-best optimal environmental taxation. Calibrated model helps to relax the restrictions of the analytical model and thus assess the economy in a more realistic setting. The calibration mostly follows the benchmark calibration we did in the first chapter with only a few changes.

7.1 Calibration

The table below summarizes the values of parameter in the calibration. The main change happens in household preference and environmental factor.

7.1.1 Household Preference

We assumed log-preference for the household as shown in Eq. 19:

$$U(c_t, M_t, 1-l_t, N_t) = \phi_1 \log c_t + \phi_2 \log M_t + (1-\phi_1-\phi_2)\log(1-l_t) + \phi_3 \log(\bar{N} - N_t) + \phi_4 \log Q_t$$

where every period, household gains utility from consumption c_t, driving M_t and leisure $1 - l_t$. Household also benefits from environmental quality Q_t and suffer from congestion N_t. Parameter Φ_1 and Φ_2 are calibrated to 0.34 and 0.05 following Wei (2013) to match the fraction of time spent on market activities.

How households value environmental quality is mostly geographically determined. We set the benchmark value to 1 to match with the city center scenario (Jackson, 1983).

Table 1: Calibration

Category	Parameters Description	Notation	Value
Driving Service	Vehicle leftover rate	ρ	0.9
	Vehicle preference	σ	0.5
	Mileage production technology	γ	0.5
Production Technology	Capital depreciation rate	ϵ_k	0.1
	Capital share in production	α_1,α_2	0.33/0.42
	Productivity level	A_1, A_2	1
	Marginal Transformation rate	μ	1
	Gasoline price	p_t	1.0872
Household Preference	Subjective discount rate	β	0.97
	Weight on consumption	ϕ_1	0.34
	Weight on driving	ϕ_2	0.05
	Weight on environmental quality	ϕ_4	1
	Marginal cost of congestion	ϕ_3	0.0127
Environmental Factor	Natural purifying capacity	ϵ	0.01
	Initial stock of environmental quality	ϕ	10
	Congestion Extreme	$\bar{N}$	1

Congestion arises because additional vehicles reduce the speed of other vehicles, and hence increase households' travel time. The average driving miles for each household is normally and average driving speed is a constant given the road condition is fairly good. Therefore, an increase in aggregate vehicle miles of travel implies more congestion. The marginal cost of congestion to household is measured by ϕ_3. Based on Newbery (1990), we calibrate the congestion cost to 0.0127 [2].

7.1.2 Environmental Factor

Environmental quality, as shown in Eq.6, is a stock variable which changes overtime based on the pollution caused by vehicle driving.ϵ measures the natural pollutant-absorbing ability and we set it to 0.01. Φ denotes the beginning level of environmental quality and we set it to 10.

7.2 Optimal Environment Tax in First Best

As shown in Eq.32 and 33, gasoline is involved in generating both type of externalities: pollution and congestion. Old cars should be taxed more for generating more pollution while new cars should be taxed more on congestion. We start from the benchmark calibration where household do not get affected by externalities ($\phi_3 = 0$,

[2] The formula for estimating marginal congestion cost come from Department of Transport.

$\phi_4 = 0$). We then change the preference value of congestion (ϕ_3) and environmental quality (ϕ_4) to see its impact on optimal tax rates.

Benchmark: no externality ($\phi_3 = 0, \phi_4 = 0$)					
Economy in Steady State					
Variable	**Description**	**Value**	**Variable**	**Description**	**Value**
c	consumption	0.3676	a	vehicle capital	0.1508
g_1	gasoline (new cars)	0.0255	δ	vehicle efficiency	0.1508
g_2	gasoline (old cars)	0.0242	l_a	labour (vehicle production)	0.0039
k_g	capital (general production)	1.4770	l_g	labour(general production)	0.3716
k_a	capital (vehicle production)	0.1657	l	total labour	0.3755
k	total capital	1.6427	P	Pollution	0.3474
Optimal Environmental Taxation					
τ^1	optimal fuel tax (new cars)	0	τ^2	optimal fuel tax (old cars)	0
T_1	optimal road tax (new cars)	0	T_2	optimal road tax (old cars)	0
Mileage of Travel					
m_1	mileage travel by new cars			0.0039	
m_2	mileage travel by old cars			0.0035	
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost for new cars			0.0277	
$(p_t + \tau^2)g_2$	gasoline cost for old cars			0.0263	
$T_1(a\delta)$	road tax cost for new cars			0	
$T_2(a\delta)$	road tax cost for old cars			0	
q^a	vehicle price			1.1353	
$q^a a\delta$	vehicle purchase cost			0.0258	

Table 2: optimal environmental tax and economy in steady state: benchmark calibration

Table 2 shows that when households do not care about externalities, optimal fuel tax and road tax are zero. Households use new cars more often and new cars provide higher mileage of travel. Households do not pay any road tax and only pay for gasoline at its original price.

Table 3 shows the economy when households only care about pollution externality ($\phi_3 = 0, \phi_4 = 0.34$). Road tax is still zero as congestion does not concern households. As households only care about pollution, new cars have a higher pollution mitigating ability than old cars and thus fuel tax rate is lower for new cars. Compared to the scenario where households ignore externalities (Table 2), gasoline consumption decrease for both types of vehicles but to different extent. New cars' gasoline consumption decreases by 9% while old cars' gasoline consumption decreases by 11%. Fuel efficiency and vehicle capital increases and pollution decreases. Households still prefer to use new cars than old ones.

We then have a look at the scenario where we have congestion but households' preference for environmental quality (ϕ_4) varies. We first evaluate the impact of environment preference on economy in steady state. Figure 1 shows how economic variables change in steady state given different preferences on environmental quality. As households value environment more and more, households reduce their usage

Scenario 2: pollution externality only ($\phi_3 = 0$, $\phi_4 = 0.34$)					
Economy in Steady State					
Variable	**Description**	**Value**	**Variable**	**Description**	**Value**
c	consumption	0.3694	a	vehicle capital	0.1512
g_1	gasoline (new cars)	0.0232	δ	vehicle efficiency	0.1512
g_2	gasoline (old cars)	0.0215	l_a	labour (vehicle production)	0.0039
k_g	capital (general production)	1.4649	l_g	labour (general production)	0.3686
k_a	capital (vehicle production)	0.1665	l	total labour	0.3725
k	total capital	1.6315	P	Pollution	0.3117
Optimal Environmental Taxation					
τ^1	optimal fuel tax (new cars)	0.1205	τ^2	optimal fuel tax (old cars)	0.1339
T_1	optimal road tax (new cars)	0	T_2	optimal road tax (old cars)	0
Mileage of Travel					
m_1	mileage travel by new cars			0.0035	
m_2	mileage travel by old cars			0.0031	
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost for new cars			0.0280	
$(p_t + \tau^2)g_2$	gasoline cost for old cars			0.0263	
$T_1(a\delta)$	road tax cost for new cars			0	
$T_2(a\delta)$	road tax cost for old cars			0	
q^a	vehicle price			1.1353	
$q^a a\delta$	vehicle purchase cost			0.0260	

Table 3: optimal environmental tax and economy in steady state: pollution externality only

of vehicles and switch their demands to consumption and leisure. Pollution keeps decreasing and environment gets improved. Fuel efficiency and vehicle capital keeps increasing as well.

Figure 2 shows the optimal fuel tax and road tax when ϕ_4 varies. Fuel tax, as we discussed before in the analytical solution, depends on the contradicting powers:marginal cost of pollution and marginal cost of congestion caused by fuel consumption. New cars are more environmentally friendly but also generate more congestion for being more efficient in providing mileage of travel. Numerical simulation suggests that the pollution mitigation ability outweighs the congestion cost when households start to care about environment. Old cars, therefore, are facing higher fuel tax than new cars. New cars provide more mileage of travel to households which implies more congestion thus road tax is higher for new cars than old cars.

7.3 Uniform Tax

Levying different gasoline tax based on vehicle type is not feasible in practice, we therefore solve for uniform fuel tax under the fuel consumption ratio between new cars and old cars (Eq.47). We obtain the solution to social planner's problem under the constraint of gasoline consumption ratio (Eq.40). In steady state, the constraint

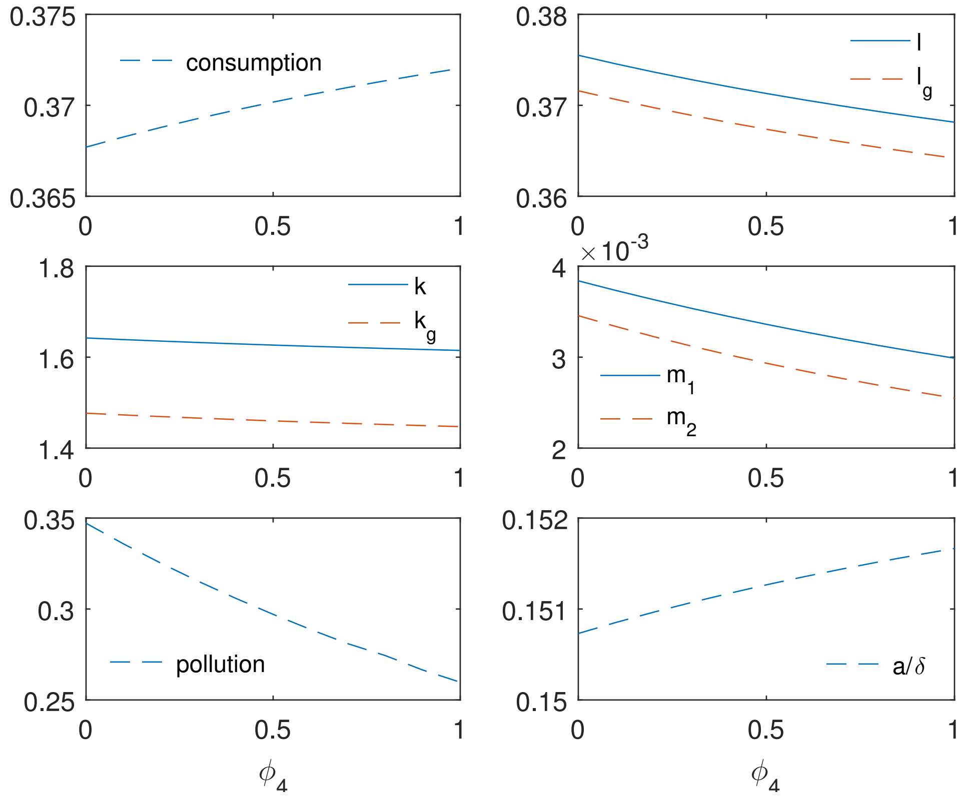


Figure 1: The impact of preference on environment

reduces to:

$$g_1 = \rho^{\frac{\gamma\sigma}{\sigma-1}} g_2 \tag{53}$$

Table 4 shows the economy in steady state with changing preference for environment quality. The gasoline consumption ratio constraint is very close to the optimal tax scenario, thus when the preference for environment varies, the change of economic variables in steady state follows the same pattern.

As weight on environment increases, households choose to drive less of both types of vehicles which leads to decreasing gasoline consumption. Households, at the same time, switch their demand to consumption and leisure to main their utility level. Pollution decreases and environment gets improved. The uniform fuel tax and corresponding road tax are shown in Figure 3:

As shown in Eq.47, uniform fuel tax takes the form of weighted average of fuel tax for new cars and old cars and thus uniform tax should lay in between those two fuel taxes. Figure 3 shows that as preference for environment grows, uniform fuel taxes increases and lies between fuel tax for new cars and old cars. Road taxes, based on calibration, are the same for both new cars and old cars and keep decreasing when households value environment more.

Scenario 3: pollution and congestion (ϕ_4 varies)

Variable	Description	Value			
		$\phi_4 = 0$	$\phi_4 = 0.1$	$\phi_4 = 0.34$	$\phi_4 = 1$
Economy in Steady State					
c	consumption	0.3677	0.3682	0.3694	0.3720
g_1	gasoline (new cars)	0.0254	0.0247	0.0231	0.0197
g_2	gasoline (old cars)	0.0241	0.0233	0.0215	0.0177
k_g	capital (general production)	1.4767	1.4729	1.4647	1.4472
k_a	capital (vehicle production)	0.1654	0.1657	0.1663	0.1675
k	total capital	1.6422	1.6387	1.6310	1.6148
a	vehicle capital	0.1507	0.1508	0.1511	0.1516
δ	vehicle efficiency	0.1507	0.1508	0.1511	0.1516
l_a	labour (vehicle production)	0.003906	0.003913	0.003926	0.003955
l_g	labour (general production)	0.3716	0.3706	0.3685	0.3641
l	total labour	0.3755	0.3745	0.3725	0.3681
P	pollution	0.3471	0.3358	0.3114	0.2597
Optimal Environmental Taxation					
τ^1	optimal fuel tax (new cars)	0.0021	0.0376	0.1227	0.3561
τ^2	optimal fuel tax (old cars)	0.0020	0.0414	0.1360	0.3953
T_1	optimal road tax (new cars)	0.0011691	0.001136	0.001064	9.0788e-04
T_2	optimal road tax (old cars)	0.0011692	0.001128	0.001041	8.6048e-04
Mileage of Travel					
m_1	mileage travel (new cars)	0.0038	0.0037	0.0035	0.0030
m_2	mileage travel (old cars)	0.0035	0.0033	0.0031	0.0025
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost for new cars	0.0277	0.0278	0.0280	0.0284
$(p_t + \tau^2)g_2$	gasoline cost for old cars	0.0263	0.0263	0.0263	0.0263
$T_1(a\delta)$	road tax cost for new cars	2.6562e-05	2.5861e-05	2.4318e-05	2.0884e-05
$T_2(a\delta)$	road tax cost for old cars	2.6566e-05	2.5685e-05	2.3792e-05	1.9793e-05
q^a	vehicle price	1.135329	1.135328	1.135334	1.135340
$q^a a\delta$	vehicle purchase cost	0.025794	0.025835	0.025925	0.02611

7.4 Welfare Analysis

In this section, we compare the welfare status under optimal tax and uniform tax. The difference between optimal fuel tax for different types of vehicles and uniform fuel tax is that we impose the gasoline consumption ratio to solve for uniform tax. As we mentioned above, given that the gasoline ratio constraint is quite close to what we have in optimal fuel tax scenario, we do not observe huge differences in economy in the long-run, which means that the welfare do not vary too much.

As shown in Figure **??**, Households are better off under optimal fuel tax but not to a large extent. As preference for environmental quality increases, the utility difference gap between two policy options becomes wider. We also solve for the consumption equivalence for one percentage improvement in environmental quality.

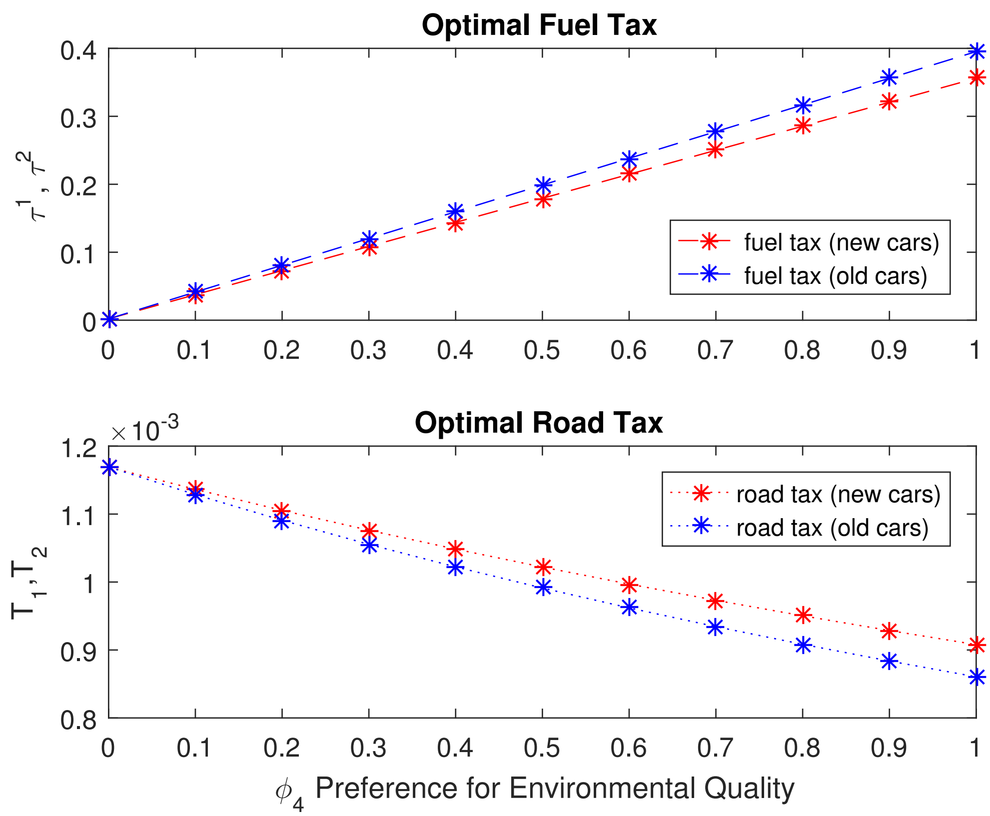


Figure 2

Households gain utility from consumption c, driving service M, leisure $1-l$ and environmental quality Q while suffer from congestion externality N. The percentage change of consumption $\frac{dc}{c}$ is expressed as:

$$\frac{dc}{c} = -\frac{\phi_4}{\phi_1}\frac{dQ}{Q} \tag{54}$$

and it depends on the preference ratio between consumption and environmental quality. We look at the equivalent percentage change of consumption when environmental quality changes:

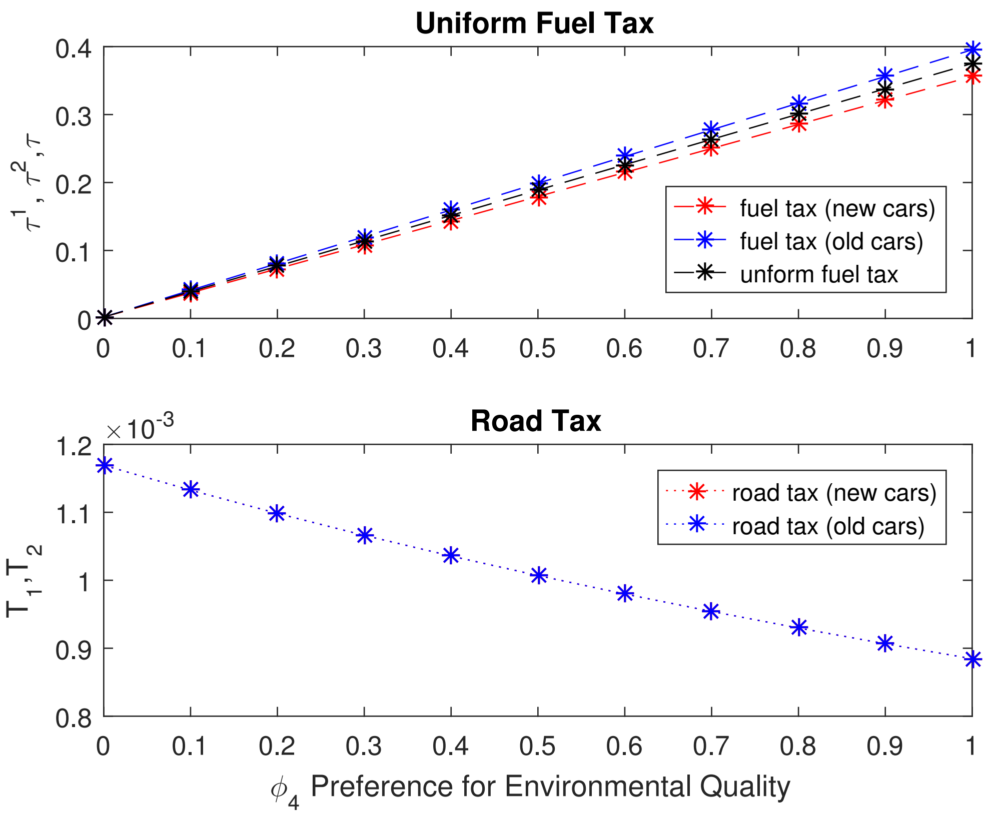
8 Conclusion

Uniform Tax: pollution and congestion (ϕ_4 varies)					
Economy in Steady State					
Variable	**Description**	**Value**			
		$\phi_4 = 0$	$\phi_4 = 0.1$	$\phi_4 = 0.34$	$\phi_4 = 1$
c	consumption	0.3677	0.3682	0.3694	0.3720
g_1	gasoline (new cars)	0.0254	0.0246	0.0229	0.0191
g_2	gasoline (old cars)	0.0241	0.0234	0.0217	0.0182
k_g	capital (general production)	1.4767	1.4729	1.4647	1.4472
k_a	capital (vehicle production)	0.1654	0.1657	0.1663	0.1675
k	total capital	1.6422	1.6386	1.6310	1.6147
a	vehicle capital	0.1507	0.1508	0.1511	0.1516
δ	vehicle efficiency	0.1507	0.1508	0.1511	0.1516
l_a	labour (vehicle production)	0.003906	0.003913	0.003926	0.003954
l_g	labour (general production)	0.3716	0.3706	0.3685	0.3641
l	total labour	0.3755	0.3745	0.3725	0.3681
P	pollution	0.3471	0.3358	0.3116	0.2600
Optimal Environmental Taxation					
τ	optimal fuel tax (new cars)	0.0020	0.0395	0.1292	0.3752
T_1	optimal road tax (new cars)	0.0012	0.00113	0.00105	8.8456e-04
T_2	optimal road tax (old cars)	0.0012	0.00113	0.00105	8.8456e-04
Mileage of Travel					
m_1	mileage travel(new cars)	0.0038	0.0037	0.0035	0.0029
m_2	mileage travel (old cars)	0.0035	0.0033	0.0031	0.0026
Travel Cost					
$(p_t + \tau^1)g_1$	gasoline cost (new cars)	0.0277	0.0278	0.0279	0.0281
$(p_t + \tau^2)g_2$	gasoline cost (old cars)	0.0263	0.0264	0.0265	0.0266
$T_1(a\delta)$	road tax cost (new cars)	2.6564e-05	2.5774e-05	2.4059e-05	2.0343e-05
$T_2(a\delta)$	road tax cost (old cars)	2.6564e-05	2.5774e-05	2.4059e-05	2.0343e-05
q^a	vehicle price	1.13533	1.135329	1.135337	1.135342
$q^a a\delta$	vehicle purchase cost	0.025794	0.025835	0.025923	0.02611

Table 4: uniform fuel tax and corresponding road tax with both types of externalities

Consumption percentage change

	$\phi_4 = 0.1$	$\phi_4 = 0.34$	$\phi_4 = 1$
dQ/Q	6.2085e-06	1.7547e-05	3.08e-05
dc/c	-1.8260e-06	-1.7547e-05	-9.0591e-05



Uniform Fuel Tax
τ^1, τ^2, τ
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
fuel tax (new cars)
fuel tax (old cars)
unform fuel tax
Road Tax
$\times 10^{-3}$
T_1, T_2
1.2
1.1
1
0.9
0.8
road tax (new cars)
road tax (old cars)
ϕ_4 Preference for Environmental Quality

Figure 3

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A Appendix:Constrained Social Planner's Problem

We need to guarantee that the first order conditions and envelope conditions measure the same marginal changes for social planner under the gasoline constraint. We set up a constrained social planner problem to see whether the marginal changes match with what we come up with above.

The objective function for social planner is the same with 7:

$$V^t(k_t, Q_t; a_{t-1}\delta_{t-1}, \rho a_{t-2}\delta_{t-2}; \left\{I_t\right\}) =$$
$$\max_{c_t, g_{t,1}, g_{t,2}, a_t, l_t^g, l_t, k_t^g, k_t} [U(c_t, M_t, 1-l_t, N_t, Q_t) + \beta V^{t+1}(k_{t+1}, Q_{t+1}; a_t\delta_t, \rho a_{t-1}\delta_{t-1}; \left\{I_{t+1}\right\})]$$

subject to:

$$\begin{aligned}
G(k_t^g, l_t^g) &= c_t + k_{t+1} - (1-\epsilon_k)k_t + p_t(g_{t,1} + g_{t,2}) \\
F(k_t^a, l_t^a) &= a_t + \mu\delta_t \\
Q_{t+1} - Q_t &= \Phi - \epsilon Q_t - P_t \\
g_{t,1} &= \Psi(a_{t-1}\delta_{t-1}, \rho a_{t-1}\delta_{t-2}) g_{t,2}
\end{aligned}$$

Thus, the corresponding first-order conditions are:

$$c_t: \quad U_{c_t} = \beta V^{t+1}_{k_{t+1}}$$

$$g_{t,2}: \quad U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial g_{t,2}} + U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial g_{t,2}}$$
$$+U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial g_{t,2}} = \beta\left[V^{t+1}_{k_{t+1}} p_t(\Psi+1) + V^{t+1}_{Q_{t+1}}(\frac{\partial P_t}{\partial g_{t,2}} + \frac{\partial P_t}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial g_{t,2}})\right]$$

$$a_t: \quad \beta V^{t+1}_{a_t\delta_t}(\delta_t + a_t\frac{\partial H}{\partial a_t}) = 0$$

$$l_t^g: \quad \beta\left[V^{t+1}_{k_{t+1}}\frac{\partial G}{\partial l_t^g} - V^{t+1}_{a_t\delta_t} a_t \frac{\partial H}{\partial (l_t - l_t^g)}\right] = 0$$

$$l_t: \quad U_{1-l_t} = \beta V^{t+1}_{a_t\delta_t} a_t \frac{\partial H}{\partial (l_t - l_t^g)}$$

$$k_t^g: \quad \beta V^{t+1}_{k_{t+1}}\frac{\partial G}{\partial k_t^g} - \beta V^{t+1}_{a_t\delta_t} a_t \frac{\partial H}{\partial (k_t - k_t^g)} = 0$$

And envelope conditions:

$$V_{k_t}^t = \beta V_{k_{t+1}}^{t+1}(1-\epsilon_k) + \beta V_{a_t\delta_t} a_t \frac{\partial H}{\partial(k_t - k_t^g)}$$

$$V_{Q_t}^t = U_{Q_t} + \beta V_{Q_{t+1}}^{t+1}(1+\epsilon)$$

$$V_{(a_{t-1}\delta_{t-1})}^t = U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(a_{t-1}\delta_{t-1})}$$
$$+U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} + \rho\beta V_{\rho a_{t-1}\delta_{t-1}}^{t+1} - \beta V_{k_{t+1}}^{t+1} p_t \frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})} - \beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(a_{t-1}\delta_{t-1})}$$

$$V_{(\rho a_{t-2}\delta_{t-2})}^t = U_{M_t}\frac{\partial M_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} + U_{M_t}\frac{\partial M_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} + U_{N_t}\frac{\partial N_t}{\partial m_{t,1}}\frac{\partial m_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}$$
$$+U_{N_t}\frac{\partial N_t}{\partial m_{t,2}}\frac{\partial m_{t,2}}{\partial(\rho a_{t-2}\delta_{t-2})} - \beta V_{k_{t+1}}^{t+1} p_t \frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})} - \beta V_{Q_{t+1}}^{t+1}\frac{\partial P_t}{\partial g_{t,1}}\frac{\partial g_{t,1}}{\partial(\rho a_{t-2}\delta_{t-2})}$$

The first order condition with respect to g_2 and envelop conditions with respect to $a_{t-1}\delta_{t-1}$, $\rho a_{t-2}\delta_{t-2}$ match with Eq. 42, 43 and 44.

B Appendix B: Steady State Solution

Equations describing the economy in steady states are:

$$A_1(k^g)^{\alpha_1}(l^g)^{1-\alpha_1} = c + \epsilon_k k + p_t(g_1 + g_2)$$
$$k^a + k^g = k$$
$$l^a + l^g = l$$
$$\mu\delta = A_2(k^a)^{\alpha_2} l^{a\frac{1}{2}-\alpha_2} - a$$
$$\epsilon Q = \Phi - P$$

and

$$V_k = \frac{U_c}{\beta} \tag{B.1}$$

$$U_M \frac{\partial M}{\partial m_1}\frac{\partial m_1}{\partial g_1} + U_N \frac{\partial N}{\partial m_1}\frac{\partial m_1}{\partial g_1} = \beta(p_t V_k + V_Q \frac{\partial P}{\partial g_1}) \tag{B.2}$$

$$U_M \frac{\partial M}{\partial m_2}\frac{\partial m_2}{\partial g_2} + U_N \frac{\partial N}{\partial m_2}\frac{\partial m_2}{\partial g_2} = \beta(p_t V_k + V_Q \frac{\partial P}{\partial g_2}) \tag{B.3}$$

$$a = \mu\delta \tag{B.4}$$

$$V_k \frac{\partial G}{\partial l^g} = V_{a\delta} a \frac{\partial H}{\partial l^a} \tag{B.5}$$

$$U_{1-l} = \beta V_{a\delta} a \frac{\partial H}{\partial l^a} \tag{B.6}$$

$$V_k \frac{\partial G}{\partial k^g} = V_{a\delta} a \frac{\partial H}{\partial k^a} \tag{B.7}$$

$$V_k = \beta V_k(1-\epsilon_k) + \beta V_{a\delta} a \frac{\partial H}{\partial k^a} \tag{B.8}$$

$$V_Q = U_Q + \beta V_Q(1+\epsilon) \tag{B.9}$$

$$V_{a\delta} = U_M \frac{\partial M}{\partial m_1}\frac{\partial m_1}{\partial(a\delta)} + U_N \frac{\partial N}{\partial m_1}\frac{\partial m_1}{\partial(a\delta)} + \rho\beta V_{\rho a\delta} \tag{B.10}$$

$$V_{\rho a\delta} = U_M \frac{\partial M}{\partial m_2}\frac{\partial m_2}{\partial(\rho a\delta)} + U_N \frac{\partial N}{\partial m_2}\frac{\partial m_2}{\partial(\rho a\delta)} \tag{B.11}$$

Using marginal substitution between consumption and capital (Eq. 55), we can get rid of V_k. Eq.59 and Eq.60 give us the marginal substitution between consumption and labour:

$$U_c \frac{\partial G}{\partial k^g} = U_{1-l}$$

Eq. 59 and Eq.61 give us the capital labour ratio between two production sectors:

$$\frac{\frac{\partial G}{\partial l^g}}{\frac{\partial G}{\partial k^g}} = \frac{\frac{\partial H}{\partial l^a}}{\frac{\partial H}{\partial l^a}}$$

Eq. 61 and Eq.62 decide the marginal productivity of labour in general production function:

$$\frac{\partial G}{\partial k^g} = \frac{1-\beta(1-\epsilon_k)}{\beta}$$

Using Eq. 63 to get rid of V_Q and Eq.62 to get the expression of $V_{a\delta}$. We get the steady state conditions:

$$A_1(\frac{k^g}{l^g})^{\alpha_1} l^g = c + \epsilon_k k + p_t(g_1 + g_2)$$

$$k^a + k^g = k$$

$$l^a + l^g = l$$

$$\mu\delta = A_2(\frac{k^a}{l^a})^{\alpha_2}(l^a)^{\frac{1}{2}} - a$$

$$\epsilon Q = \Phi - P$$

$$P = \frac{g_1}{\delta} + \frac{g_2}{\rho\delta}$$

$$\frac{\Phi_2 g_1^{\sigma-1}}{g_1^\sigma + \rho^{\gamma\sigma} g_2^\sigma} + \frac{\Phi_3}{g_1 + \rho^\gamma g_2} = \frac{p_t \Phi_1}{c} + \frac{\beta}{1-\beta(1+\epsilon)} \frac{1}{\delta} \frac{\Phi_4}{Q}$$

$$\frac{\Phi_2 \rho^{\gamma\sigma} g_2^{\sigma-1}}{g_1^\sigma + \rho^{\gamma\sigma} g_2^\sigma} + \frac{\Phi_3 \rho^\gamma}{g_1 + \rho^\gamma g_2} = \frac{p_t \Phi_1}{c} + \frac{\beta}{1-\beta(1+\epsilon)} \frac{1}{\rho\delta} \frac{\Phi_4}{Q}$$

$$a = \mu\delta$$

$$(\frac{k^g}{l^g})^{\alpha_1} = \frac{1-\Phi_1-\Phi_2}{\Phi_1 A_1(1-\alpha_1)} \frac{c}{1-l}$$

$$\frac{k^a}{l^a} = \frac{1-\alpha_1}{\alpha_1} \frac{\alpha_2}{\frac{1}{2}-\alpha_2} \frac{k^g}{l^g}$$

$$\frac{k^g}{l^g} = \left[\frac{1-\beta(1-\epsilon_k)}{\beta A_1 \alpha_1}\right]^{\frac{1}{\alpha_1 - 1}}$$

$$\frac{[1-\beta(1-\epsilon_k)]\mu}{\beta A_2 \alpha_2 (\frac{1-\alpha_1}{\alpha_1})^{\alpha_2-\frac{1}{2}} (\frac{\alpha_2}{\frac{1}{2}-\alpha_2})^{\alpha_2-\frac{1}{2}} (\frac{1-\beta(1-\epsilon_k)}{\beta A_1 \alpha_1})^{\frac{\alpha_2-\frac{1}{2}}{\alpha_1-1}} (k^a)^{-\frac{1}{2}}} \frac{\Phi_1}{\beta c} = \frac{p_t \Phi_1 (g_1 + \beta g_2)}{c} + \frac{\beta \Phi_4 (g_1 + \beta g_2)}{(1-\beta(1+\epsilon))\delta Q}$$